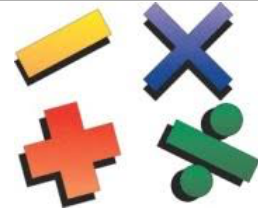


Date: _____

Unit 1: Number Sense

Lesson 2: Operations with Rational Numbers



1) Vocabulary

What is a rational number? Provide some examples.

any positive or negative fraction

What does a fraction represent?

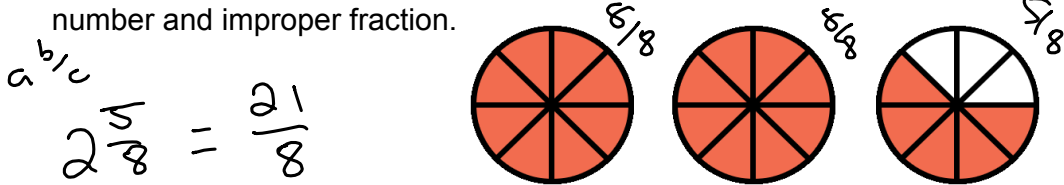
part of a whole

What is an improper fraction? A mixed number?

numerator is larger than denominator (5/2) a proper fraction together (2 1/2) → a whole and

2) Converting Mixed Numbers to Improper Fractions

The diagram below shows a fraction greater than one. Write both a mixed number and improper fraction.



How can we change a mixed number to an improper fraction without a diagram?

multiply the whole number with the denominator, then add the numerator
leave the bottom the same $\frac{2 \times 8 + 5}{8} = \frac{21}{8}$
Write each of the following as an improper fraction. Show your work!

$3\frac{5}{7}$	$-3\frac{5}{7}$	$12\frac{12}{13}$	$-1\frac{3}{25}$
$= \frac{3 \times 7 + 5}{7}$	$= -\left(\frac{3 \times 7 + 5}{7}\right)$	$= \frac{12 \times 13 + 12}{13}$	$= -\left(\frac{1 \times 25 + 3}{25}\right)$
$= \frac{26}{7}$	$= -\frac{26}{7}$	$= \frac{168}{13}$	$= -\frac{28}{25}$

Why do we need to be careful with negative whole numbers?



The whole expression is negative in that case, so we need to leave the negative in front while we multiply.

$$-3\frac{5}{7} + \frac{2}{7}$$

$$= -\frac{21}{7} + \frac{2}{7}$$

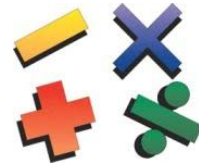
$$= \frac{-21+2}{7}$$

$$= -\frac{19}{7}$$

2) Multiplying and Dividing Rational Numbers

How do we multiply fractions?

- multiply across the top + bottom



Find the product for each of the following. Write your answer in lowest terms.

$$\begin{array}{l} \frac{4}{5} \times \frac{9}{7} \\ = \frac{4 \times 9}{5 \times 7} \\ = \frac{36}{35} \end{array} \qquad \begin{array}{l} \frac{3}{8} \times \left(-\frac{4}{5}\right) \\ = -\frac{12}{40} \\ = -\frac{3}{10} \end{array} \qquad \begin{array}{l} \left(-\frac{2}{3}\right) \left(-\frac{11}{9}\right) \\ = \frac{22}{27} \end{array}$$

Cross cancelling allows us to reduce to lowest terms earlier, and work with more manageable numbers. Please use this skill when it is appropriate!

$$\begin{array}{l} \frac{5 \cancel{4}^1 \times 9}{2 \cancel{24}^3 \times 27 \cdot 3} = \frac{5}{2} \times \frac{1}{3} \\ = \frac{5}{6} \end{array}$$

What do we need to do to divide fractions?

- keep the first fraction like it is
- flip the second one and multiply

Find the quotient for each of the following. Write your answer in lowest terms.

$$\begin{array}{l} \left(2\frac{3}{4}\right) \div \left(1\frac{8}{9}\right) \\ = \left(\frac{11}{4}\right) \div \left(\frac{17}{9}\right) \\ = \frac{11}{4} \times \frac{9}{17} \\ = \frac{99}{68} \end{array} \qquad \begin{array}{l} \left(\frac{11}{12}\right) \div \left(-\frac{5}{6}\right) \\ = \frac{11}{12} \times \left(-\frac{6}{5}\right) \\ = -\frac{11}{10} \end{array}$$

3) Adding and Subtracting Rational Numbers

a) Finding a Common Denominator

We cannot add or subtract any fractions before they have a common denominator. Remember, to find a common denominator, we just need to use a common multiple for all terms. For example, if one fraction has a denominator of 3 and another has a denominator of 4, what would be a good common denominator to use?

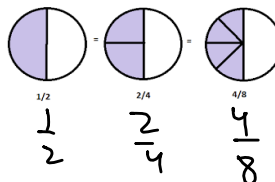
$$\text{ex) } \frac{1}{3} + \frac{1}{4} \quad \text{C.D.} = 12$$

Write each set of fractions below with a common denominator.

$CD = 21$ $\frac{-5 \times 3}{7 \times 3}$ and $\frac{2 \times 7}{3 \times 7}$ $\frac{-15}{21}$ and $\frac{14}{21}$ $\frac{3 \times 12}{2 \times 12}$, $\frac{1 \times 3}{3 \times 3}$, and $\frac{7 \times 2}{12 \times 2}$ $CD = 24$
 $\frac{36}{24}$, $\frac{21}{24}$, $\frac{14}{24}$

What is an equivalent fraction? It is important to check that you have created equivalent fractions when you find a common denominator!

fractions that represent the same part of a whole.



b) Adding and Subtracting

To add/subtract rational numbers, you need to express all terms with a common denominator. Then you can apply integer rules (combine double signs!) and add/subtract the numerators. The denominator stays the same!! Reduce to lowest terms if necessary.

Find the sum/difference for each of the following. Show your work!

i) $\frac{5}{7} + \left(\frac{2}{3}\right)$ $CD = 21$

$$= \frac{5 \times 3}{7 \times 3} + \frac{2 \times 7}{3 \times 7}$$

$$= \frac{15}{21} + \frac{14}{21}$$

$$= \frac{29}{21}$$

ii) $\left(-\frac{4}{5}\right) + \left(-\frac{3}{10}\right)$ $CD = 10$

$$= -\frac{4 \times 2}{5 \times 2} - \frac{3}{10}$$

$$= -\frac{8}{10} - \frac{3}{10}$$

$$= -\frac{11}{10}$$

iii) $-\frac{5}{2} + \left(-\frac{1}{5}\right) - \left(-3\right)$ $CD = 10$

$$= -\frac{5 \times 5}{2 \times 5} - \frac{1 \times 2}{5 \times 2} + \frac{3 \times 10}{1 \times 10}$$

$$= -\frac{25}{10} - \frac{2}{10} + \frac{30}{10}$$

$$= \frac{-25 - 2 + 30}{10}$$

$$= \frac{3}{10}$$



4) Applying the Math

$2\frac{1}{2}$ batches.


A recipe is shown below. You want to make two and half batches of cookies. How much of each ingredient do you need? Show your work!

Crispy Coconut-Oatmeal Cookies Makes 6 servings

1/2 cup solid vegetable shortening	1 3/4 cups flour
1/2 cup margarine	1 teaspoon baking powder
1 cup packed brown sugar	1 teaspoon baking soda
1 cup sugar	2 cups coarsely chopped nuts
2 eggs	1 cup oats
1 teaspoon vanilla extract	1 cup coconuts

Cream shortening, margarine, and sugars. Beat in eggs and vanilla. Combine flour, baking powder, and baking soda, then beat into creamed mixture. Stir in nuts, oats, and coconut. Let stand 15 minutes.

Preheat oven to 400°. Drop by teaspoonfuls onto ungreased cookie sheets. Bake 8 to 10 minutes or until golden. Makes about 6 dozen.



Handwritten notes on the recipe: $2\frac{1}{2}$ cups (next to brown sugar), $2\frac{1}{2}$ cups (next to sugar), $2\frac{1}{2}$ tsp (next to vanilla extract), $2\frac{1}{2}$ tsp (next to baking powder), $2\frac{1}{2}$ tsp (next to baking soda), $2\frac{1}{2}$ cups (next to flour), $2\frac{1}{2}$ cups (next to oats), $2\frac{1}{2}$ cups (next to coconuts), and 5 cups (next to nuts).

1 batch

$\frac{1}{2}$ cup shortening

$\frac{1}{2}$ cup margarine

$1\frac{3}{4}$ cup flour

$2\frac{1}{2}$ batches

$$\frac{1}{2} \times \frac{5}{2} = \frac{5}{4} \text{ or } 1\frac{1}{4}$$

$$\frac{1}{2} \times \frac{5}{2} = \frac{5}{4} \text{ or } 1\frac{1}{4}$$

$$\frac{7}{4} \times \frac{5}{2} = \frac{35}{8} \text{ or } 4\frac{3}{8}$$

A friend has already made these cookies and suggests that you add an extra $\frac{2}{3}$ cup of oats to each batch. How many cups of oats should you add to your blend?

1 batch originally had 1 cup of oats.

$$1\frac{2}{3} \text{ cups now} \rightarrow \frac{5}{3} \times \frac{5}{2} = \frac{25}{6} \text{ or } 4\frac{1}{6}$$

