Chapter 4: Factoring Algebraic Expressions
Getting Started

1a) binomial → $2x-5$ (i)
b) coefficient → $\boxed{8}x^2$ (iv)
c) factoring → $24 = 2\times2\times2\times3$ (v)
d) monomial → $4xy$ (vi)
e) variable → x (vii)
f) trinomial → $7x^2+3x-1$ (iii)
g) expanding → $4(2x-3) = 8x-12$ (ii)
h) like terms → $6y + -8y$ (viii)

2a) $4x-6y+8y-5x$
$= -x+2y$
b) $5ab-6a^2+6ab-3a^2-11ab+9b^2$
$= -9a^2-6ab+9b^2$
c) $(2x-5y)+(7x+4)-(5x-y)$
$= 2x-5y+7x+4-5x+y$
$= 4x-4y+4$
d) $(7a-2ab)-(4b+5a)+(ab-3a)$
$= 7a-2ab-4b-5a+ab-3a$
$= -a-ab-4b$

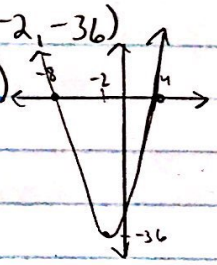
3a) $7(2x-5)$
$= 14x-35$
b) $-5x(3x^2-4x+5)$
$= -15x^3+20x^2-25x$
c) $2(4x^2+3x+1)-2x(8x-3)$
$= 8x^2+6x+2-16x^2+6x$
$= -8x^2+12x+2$
d) $(d-6)(d+2)$
$= d^2+2d-6d-12$
$= d^2-4d-12$
e) $(3a-7b)(4a-3b)$
$= 12a^2-9ab-28ab+21b^2$
$= 12a^2-37ab+21b^2$
f) $6x(2x+1)^2$
$= 6x(2x+1)(2x+1)$
$= 6x(4x^2+2x+2x+1)$
$= 6x(4x^2+4x+1)$
$= 24x^3+24x^2+6x$

4a) $4(2x+5)$
$= 8x+20$
b) $2(2x^2+2x)$
$= 4x^2+4x$
c) $(x+2)(3x+2)$
$= 3x^2+8x+4$
d) $(x-2)(3x+2)$
$= 3x^2-4x-4$

5a) $(x^5)(x^7)$
$= x^{12}$
b) $(-6a^2)(3a^4)(2a)$
$= -36a^7$
c) $(4y)(3y^2)+2y^3$
$= 12y^3+2y^3$
$= 14y^3$
d) $20z^5 \div (-4z^3)(-z^2)$
$= (-5z^2)(-z^2)$
$= 5z^4$

6a) $5x$
b) $2x+4$
c) x^2+2x+3
d) x^2+4x+3

7a) $28 \to 1, 2, 4, \textcircled{7}, 14, 28$
$35 \to 1, 5, \textcircled{7}, 35$
GCF: 7
b) $36 \to 1, 2, \textcircled{3}, 4, 6, 12, 18, 36$
$63 \to 1, \textcircled{3}, 7, 9, 21, 63$
GCF: 3
c) $99 \to 1, 3, \textcircled{9}, 11, 33, 99$
$90 \to 1, 2, 3, 5, \textcircled{9}, 10, 18, 30, 45, 90$
GCF: 9
d) $4x \to 1, 2, \textcircled{4}, x$
$8 \to 1, 2, \textcircled{4}, 8$
GCF: 4
c) $25 \to 1, \textcircled{5}, 25$
$5x^2 \to 1, \textcircled{5}, x, x$
GCF: 5
f) $12y \to 1, 2, 3, 4, \textcircled{6}, 12, y$
$6x \to 1, 2, 3, \textcircled{6}, x$
GCF: 6.

8a) $y = (x-4)(x+8)$
x-int: $(4, 0)$ & $(-8, 0)$
aofs: $x = -2$
vertex: $y = (-2-4)(-2+8)$
$= -36$
$(-2, -36)$
b)


8c) $y = x^2+8x-4x-32$
$= x^2+4x-32$

10a) $x+3 \to$ iii
b) $-3x+2 \to$ ii
c) $x^2-1 \to$ i

12a) True → even #'s are divisible by 2
b) True → $(6x^2)(y^3) = 6x^2y^3$
c) False → 100: 1×100, 2×50, 4×25, 5×20, 10×10.

4.1 Common Factors in Polynomials

1a) $4x^2 - 12x$
GCF: $4x$

b) $-6x^2 + 12x$
GCF: $-6x$

2a) $4x + 16$
GCF: 4

b) $6x^2 + 8x$
GCF: $2x$

3a) $6x$ and $10x$
GCF: $2x$

b) $15a^3$ and $20a^2$
GCF: $5a^2$

c) ab and a^2b^2
GCF: ab

d) $-2x^4y^4$ and $8x^3y^5$
GCF: $2x^3y^4$

4a) $3x^2 - 9x + 12$
GCF: 3

b) $5x^2 + 3x$
GCF: x

c) $x^2y - xy^2$
GCF: xy

d) $4x(x-1) + 3(x-1)$
GCF: $x-1$

5a) $8xy$
$= 4(2xy)$

b) $-6x^2$
$= (3x^2)(2)$

c) $15x^4z$
$= (5x^2)(3x^2z)$

d) $-49a^2b^5$
$= (-7a^2b^2)(7b^3)$

e) $-12x^3y^3$
$= (3y^3)(-4x^3)$

f) $30m^2n^3$
$= (-5m^2n)(-6n^2)$

6a) $4x - 4y$
$= 4(x-y)$

b) $8x - 2y$
$= 2(4x-y)$

c) $5a + 10b$
$= 5(a+2b)$

d) $36x^2 - 32y^3$
$= 4(9x^2 - 8y^3)$

e) $-24x^2 - 6y$
$= -6(4x^2 + y)$

f) $45a^4 - 54a^3$
$= 9a^3(5a - 6)$

7a) $7x^2 + 14x - 21$
GCF = 7

b) $3b^2 + 15b$
GCF = $3b$

c) $12c^2 - 8c + 16$
GCF = 4

d) $-25m^2 - 10m$
GCF = $-5m$

e) $3d^4 - 9d^2 + 15d^3$
GCF = $3d^2$

f) $y^3 + y^5 - y^2$
GCF = y^2

8a) $9x^2 - 6x + 18$
$= 3(3x^2 - 2x + 6)$

b) $25a^2 - 20a$
$= 5a(5a - 4)$

c) $27y^3 - 9y^4$
$= 9y^3(3 - y)$

d) $2b(b+4) + 5(b+4)$
$= (b+4)(2b+5)$

e) $4c(c-3) - 5(c-3)$
$= (c-3)(4c-5)$

f) $x(3x-5) + (3x-5)(x+1)$
$= (3x-5)(x + x + 1)$
$= (3x-5)(2x+1)$

9a) $dc^2 - 2acd + 3a^2d$
$= d(c^2 - 2ac + 3a^2)$

b) $-10a^2c + 20ac - 5ac^3$
$= -5ac(2a - 4 + c^2)$

c) $10ac^2 - 15a^2c + 25$
$= 5(2ac^2 - 3a^2c + 5)$

d) $2a^2c^4 - 4a^3c^3 + 6a^4c^2$
$= 2a^2c^2(c^2 - 2ac + 3a^2)$

e) $3a^5c^3 - 2ac^2 + 7ac$
$= ac(3a^5c^2 - 2c + 7)$

f) $10c^3d - 8cd^2 + 2cd$
$= 2cd(5c^2 - 4d + 1)$

- a) & d) have the same trinomial as one of their factors.

10a) $ax - ay + bx - by$
$= a(x-y) + b(x-y)$
$= (x-y)(a+b)$

b) $10x^2 + 5x - 6xy - 3y$
$= 5x(2x+1) - 3y(2x+1)$
$= (2x+1)(5x-3y)$

c) $3mx + 3my + 2x + 2y$
$= 3m(x+y) + 2(x+y)$
$= (x+y)(3m+2)$

d) $5my + tm + 5ny + tn$
$= m(5y+t) + n(5y+t)$
$= (m+n)(5y+t)$

e) $5wx - 10w - 3tx + 6t$
$= 5w(x-2) - 3t(x-2)$
$= (x-2)(5w-3t)$

f) $4mnt - 16mn - t + 4$
$= 4mn(t-4) - 1(t-4)$
$= (4mn-1)(t-4)$

11. $A = \frac{1}{2}(a+b)h$

$a+b = 7b+1$

$A = 70b + 10$

$h = 20$

12a) $y = 2x^2 - 10x$ ★

i) $y = 2x(x-5)$

ii) $(0,0), (5,0)$

$x = 2.5$

iii) $y = 2(\frac{5}{2})^2 - 10(\frac{5}{2})$

$= \frac{25}{2} - \frac{50}{2}$

$= -\frac{25}{2}$

$(\frac{5}{2}, -\frac{25}{2})$

iv) On grid.

b) $y = -x^2 - 8x$ ★

i) $y = -x(x+8)$

ii) $x = -4$

$(0,0), (-8,0)$

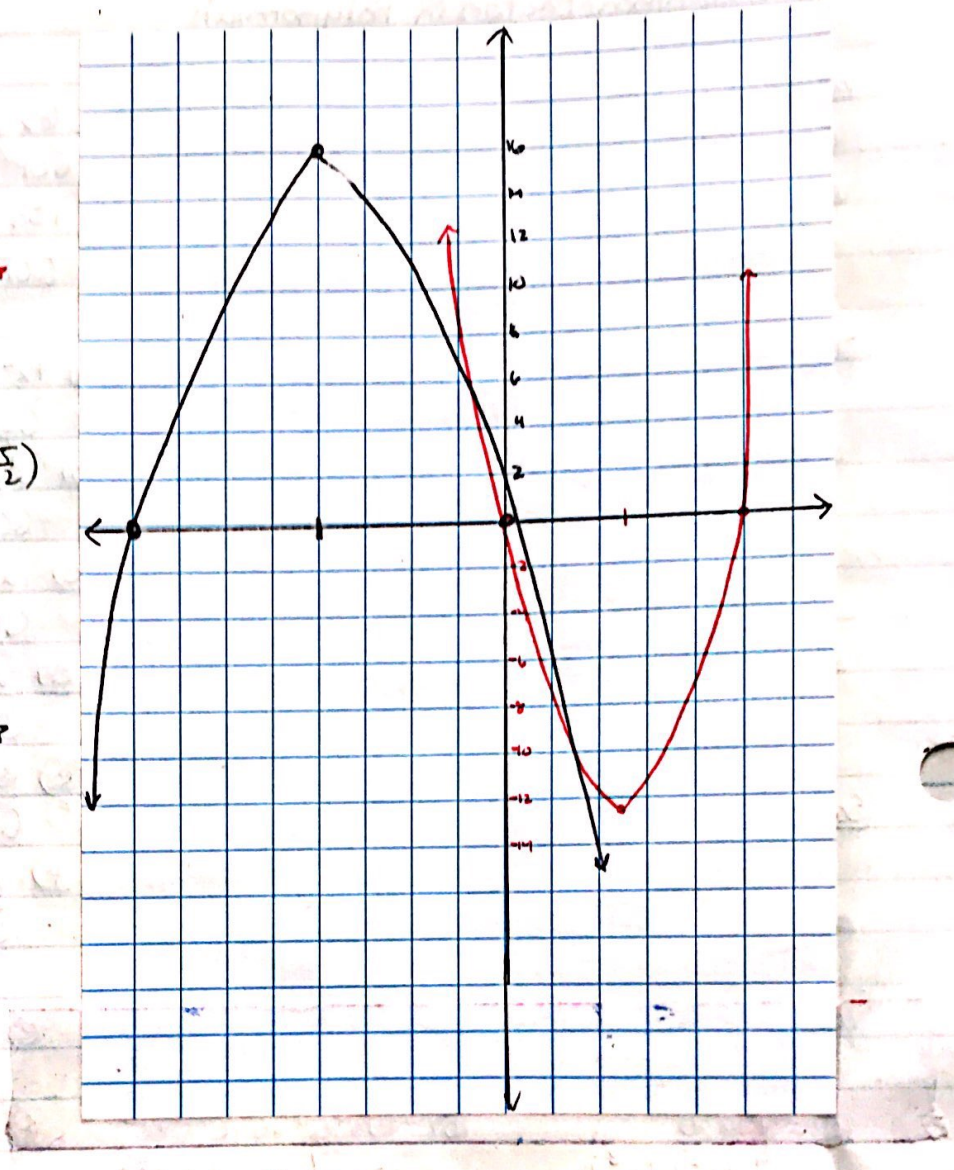
iii) $y = -(-4)^2 - 8(-4)$

$= -16 + 32$

$= 16$

$(-4, 16)$

iv) on grid.



13. $SA = 2lw + 2wh + 2lh$

$= 2(lw + wh + lh)$

14. $y = 10x - x^2$ $\quad$ $y = 3x^2 - 30x$

$y = -x(x-10)$ $\quad$ $y = 3x(x-10)$

Same: Zeros: $(0,0), (10,0)$ $\quad$ Zeros: $(0,0), (10,0)$

aofs: $x = 5$ $\quad$ aofs: $x = 5$

Vertex: $(5, 25)$ $\quad$ Vertex: $(5, -75)$

max $\rightarrow y = 25$ $\quad$ min: $y = -75$

Distance from 25 to -75:

100 units.

15a) $5x^2 - 10x$ $\quad$ $10x^2 - 15x$ $\quad$ $5x^2 - 30x$

$= 5x(x-2)$ $\quad$ $= 5x(2x-3)$ $\quad$ $= 5x(x-6)$

b) Their examples are okay - bad question.

16. They are both correct because the GCF can be $\pm$.

Let x be an integer.

17. $x^2 + (x+1)^2 + (x+2)^2 + 1$

$= x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 + 1$

$= 3x^2 + 6x + 6$

$= 3(x^2 + 2x + 3)$

↑ GCF is 3, so divisible by 3.

18. Expand it. Common factoring is the opposite of the distributive property.

19. $A_S = (2r)^2 - \pi r^2$

$= 4r^2 - \pi r^2$

$= r^2(4 - \pi)$

20a) $\frac{2x^2y + 3xy^2}{xy}$

$= \frac{xy(2x + 3y)}{xy}$

$= 2x + 3y$

b) $\frac{6x^3y + 12x^3y^2}{6x^3y}$

$= \frac{6x^3y(1 + 2y)}{6x^3y}$

$= 1 + 2y$

c) $\frac{-12x^3y^2 - 18x^2y^3}{6x^2y^2}$

$= \frac{-6x^2y^2(2x + 3y)}{6x^2y^2}$

$= -(2x + 3y)$

d) $\frac{3x^4 + 6x^3 + 9x^2}{3x^2}$

$= \frac{3x^2(x^2 + 2x + 3)}{3x^2}$

$= x^2 + 2x + 3$

4.3 Factoring Quadratics: x^2+bx+c

1a) x^2+5x+6
c) $=(x+3)(x+2)$

2a) x^2-x-6
$=(x-3)(x+2)$

b) $x^2-7x+12$
$=(x-3)(x-4)$

3a) $x^2-10x+21$
$=(x-7)(x-3)$

b) $x^2+4x-32$
$=(x-4)(x+8)$

c) $x^2-2x-63$
$=(x-9)(x+7)$

d) $x^2+14x+45$
$=(x+5)(x+9)$

4a) x^2+2x+1
$=(x+1)(x+1)$
$=(x+1)^2$

b) x^2-2x+1
$=(x-1)(x-1)$
$=(x-1)^2$

c) x^2-2x-3
$=(x-3)(x+1)$

d) x^2+6x+9
$=(x+3)(x+3)$
$=(x+3)^2$

e) x^2-4x+4
$=(x-2)(x-2)$
$=(x-2)^2$

f) $x^2-4x-12$
$=(x-6)(x+2)$

5a) $x^2+3x-10$
$=(x+5)(x-2)$

b) x^2-4x+4
$=(x-2)(x-2)$
$=(x-2)^2$

6a) $x^2+11x+24$
$=(x+3)(x+8)$

b) $c^2-15c+56$
$=(c-7)(c-8)$

c) $a^2-11a-60$
$=(a-15)(a+4)$

d) $y^2-20y-44$
$=(y+2)(y-22)$

e) $b^2+2b-48$
$=(b+8)(b-6)$

f) $z^2-19z+90$
$=(z-10)(z-9)$

7a) x^2+4x+3
$=$

b) $a^2-9a+20$
$=(a-4)(a-5)$

c) $m^2-8m+16$
$=(m-4)(m-4)$

d) n^2+n-6
$=(n+3)(n-2)$

e) $x^2+6x-16$
$=(x+8)(x-2)$

f) $x^2+15x-16$
$=(x+16)(x-1)$

8a) $x^2-10x+16$
$=(x-8)(x-2)$

b) $y^2+6y-40$
$=(y+10)(y-4)$

c) a^2-a-56
$=(a-8)(a+7)$

d) $w^2-5w-14$
$=(w-7)(w+2)$

e) $m^2-12m+32$
$=(m-8)(m-4)$

f) n^2+n-42
$=(n-6)(n+7)$

9a) $3x^2+24x+45$
$=3(x^2+8x+15)$
$=3(x+3)(x+5)$

b) $2y^2-2y-60$
$=2(y^2-y-30)$
$=2(y-6)(y+5)$

c) $3v^2+9v+6$
$=3(v^2+3v+2)$
$=3(v+2)(v+1)$

d) $6n^2+24n-30$
$=6(n^2+4n-5)$
$=6(n+5)(n-1)$

e) x^3+5x^2+4x
$=x(x^2+5x+4)$
$=x(x+4)(x+1)$

f) $7x^4+28x^3-147x^2$
$=7x^2(x^2+4x-21)$
$=7x^2(x+7)(x-3)$

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10. $(x-2)(x-2)$ $(x-2)(x+1)$ $(x-2)(x-1)$

$= x^2 - 4x + 4$ $= x^2 - x - 2$ $= x^2 - 3x + 2$

11. $x^2 - 15x + 44$ ← Martina must have done: $(11-x)(4-x)$

$= (x-4)(x-11)$ ~~These are the same~~ → $= 44 - 15x + x^2$

12a) $a^2 + 8a + 15$

$= (a+5)(a+3)$

b) $3x^2 - 21x - 54$

$= 3(x^2 - 7x - 18)$

$= 3(x-9)(x+2)$

c) $z^2 - 16z + 55$

$= (z-11)(z-5)$

d) $x^2 + 5x - 50$

$= (x+10)(x-5)$

e) $x^3 - 3x^2 - 10x$

$= x(x^2 - 3x - 10)$

$= x(x-5)(x+2)$

f) $2xy^2 - 26xy + 84x$

$= 2x(y^2 - 13y + 42)$

$= 2x(y-7)(y-6)$

13a) $y = x^2 + 2x - 8$ *

i) $(x+4)(x-2) = 0$

ii) $x = -4, x = 2$

iii) $x = -1$

$y = (-1)^2 + 2(-1) - 8$

$= -9$

$(-1, -9)$

b) $y = x^2 - 2x - 24$ *

i) $(x-6)(x+4) = 0$

ii) $x = 6, x = -4$

iii) $x = 1$

$y = (1)^2 - 2(1) - 24$

$= -25$

$(1, -25)$

c) $y = x^2 - 8x + 15$ *

i) $(x-5)(x-3) = 0$

ii) $x = 5, x = 3$

iii) $x = 4$

$y = (4)^2 - 8(4) + 15$

$= -1$

$(4, -1)$

d) $y = -x^2 - 9x - 14$ b

$y = -(x^2 + 9x + 14)$

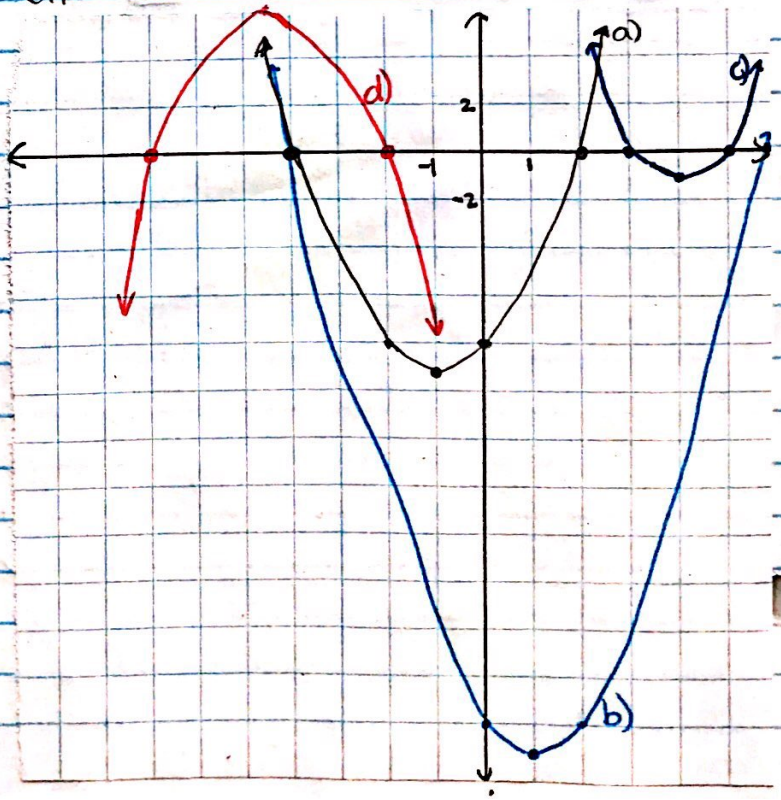
i) $-(x+7)(x+2) = 0$

ii) $x = -7, x = -2$

iii) $x = -9/2$

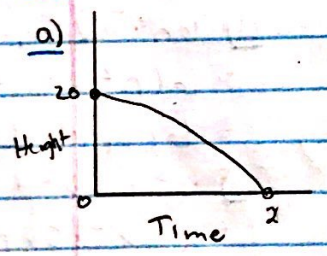
$y = -(-9/2)^2 - 9(-9/2) - 14$

$= 25/4$ $(-4\frac{1}{2}, 6\frac{1}{4})$



14. $h = -5t^2 + 20$

a)



b) Let $t = 0$

$h = 20$m

c) Let $h = 0$

$-5t^2 + 20 = 0$

$-5(t^2 - 4) = 0$

$-5(t+2)(t-2) = 0$

∴ It took 2 seconds.

15. $h = -5t^2 + 10t + 40$

① Find zeros:

$-5(t^2 - 2t - 8) = 0$

$-5(t-4)(t+2) = 0$

$t = 4,\ t = -2$

② Find vertex:

$t = 1$

$h = -5(1)^2 + 10(1) + 40$

$= 45m$

∴ The max height is 45m.

16a) $m^2 + 4mn - 5n^2$

$= (m+5n)(m-n)$

b) $x^2 + 12xy + 35y^2$

$= (x+7y)(x+5y)$

c) $a^2 + ab - 12b^2$

$= (a+4b)(a-3b)$

d) $c^2 - 12cd - 85d^2$

$= (c-17d)(c+5d)$

e) $r^2 + 13rs + 12s^2$

$= (r+12s)(r+1s)$

f) $18p^2 - 9pq + q^2$

$= q^2 - 9pq + 18p^2$

$= (q-3p)(q-6p)$

17. $x^2 + bx - c$ $\quad$ $x^2 - bx - c$

ex/ $x^2 + 2x - 3$ $\quad$ ex/ $x^2 - 2x - 3$

$= (x+3)(x-1)$ $\quad$ $= (x-3)(x+1)$

works ⇓

19a) $x^4 + 6x^2 - 27$

$= (x^2+9)(x^2-3)$

b) $a^4 + 10a^2 + 9$

$= (a^2+9)(a^2+1)$

c) $-4m^4 + 16m^2n^2 + 20n^4$

$= -4(m^4 - 4m^2n^2 - 5n^4)$

$= -4(m^2 - 5n^2)(m^2 + n^2)$

d) $(a-b)^2 - 15(a-b) + 26$

$= (a-b-13)(a-b-2)$

20a) $\dfrac{x^2 - 6x + 8}{x-4}$

$= \dfrac{(x-4)(x-2)}{x-4}$

$= x - 2$

b) $\dfrac{a^2 - 3a - 28}{a+4}$

$= \dfrac{(a-7)(a+4)}{a+4}$

$= a - 7$

c) $\dfrac{x^2 + x - 30}{x-5}$

$= \dfrac{(x+6)(x-5)}{x-5}$

$= x + 6$

d) $\dfrac{2(x^2 - 12x + 32)}{2(x-8)}$

$= \dfrac{2(x-8)(x-4)}{2(x-8)}$

$= x - 4$

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4.4 Factoring Quadratics: ax^2+bx+c

2a) $2x^2+5x+2$
$=(2x+1)(x+2)$

b) $6x^2-5x+1$
$=(3x-1)(2x-1)$

c) $8x^2+26x+15$
$=(4x+3)(2x+5)$

d) $15x^2+13x-6$
$=(5x+6)(3x-1)$

4a) $(x+3)(5x+1)$
$=5x^2+5x+15x+3$
$=5x^2+20x+3$

b) $(2x+3)(x-2)$
$=2x^2-4x+3x-6$
$=2x^2-x-6$

c) $(3x-1)(4x-1)$
$=12x^2-3x-4x+1$
$=12x^2-7x+1$

d) $(2x-3)(7x-4)$
$=14x^2-21x-8x+12$
$=14x^2-29x+12$

5a) $2x^2+x-6$ $\quad ac=-12$
$=2x^2+4x-3x-6$ $\quad b=1$
$=2x(x+2)-3(x+2)$ $\quad (4)(-3)=12$
$=(2x-3)(x+2)$ $\quad (4)+(-3)=1$

b) $3n^2-11n-4$ $\quad ac=-12$
$=3n^2-12n+1n-4$ $\quad b=-11$
$=3n(n-4)+1(n-4)$ $\quad (-12)(1)=-12$
$=(n-4)(3n+1)$ $\quad (-12)+(1)=-11$

c) $10a^2+3a-1$ $\quad ac=-10$
$=10a^2+5a-2a-1$ $\quad b=3$
$=5a(2a+1)-1(2a+1)$ $\quad (5)(-2)=-10$
$=(5a-1)(2a+1)$ $\quad (5)+(-2)=3$

3a) $2c^2+7c-4$ $\quad ac=-8$
$=2c^2+8c-1c-4$ $\quad b=7$
$=2c(c+4)-1(c+4)$ $\quad (8)(-1)=-8$
$=(c+4)(2c-1)$ $\quad (8)+(-1)=7$

b) $4z^2-9z-9$ $\quad ac=-36$
$=4z^2-12z+3z-9$ $\quad b=-9$
$=4z(z-3)+3(z-3)$ $\quad (-12)(3)=-36$
$=(z-3)(4z+3)$ $\quad (-12)+(3)=-9$

d) $6p^2+7p-3$ $\quad ac=-18$
$=6p^2+9p-2p-3$ $\quad b=7$
$=3p(2p+3)-1(2p+3)$ $\quad (9)(-2)=-18$
$=(3p-1)(2p+3)$ $\quad (9)+(-2)=7$

oops!

c) $6y^2-y-1$ $\quad ac=-6$
$=6y^2-3y+2y-1$ $\quad b=-1$
$=3y(2y-1)+1(2y-1)$ $\quad (-3)(2)=-6$
$=(3y+1)(2y-1)$ $\quad (-3)+(2)=-1$

5d) $4x^2-16x+15$ $\quad ac=60$
$=4x^2-10x-6x+15$ $\quad b=-16$
$=2x(2x-5)-3(2x-5)$ $\quad (-10)(-6)=60$
$=(2x-3)(2x-5)$ $\quad (-10)+(-6)=-16$

e) $2c^2+5c-12$ $\quad ac=-24$
$=2c^2+8c-3c-12$ $\quad b=5$
$=2c(c+4)-3(c+4)$ $\quad (8)(-3)=-24$
$=(2c-3)(c+4)$ $\quad (8)+(-3)=5$

f) $6x^2+5x+1$ $\quad ac=6$
$=6x^2+3x+2x+1$ $\quad b=5$
$=3x(2x+1)+1(2x+1)$ $\quad (3)(2)=6$
$=(3x+1)(2x+1)$ $\quad (3)+(2)=5$

Hilary

6a) $6x^2 - 13x + 6$ $ac = 36$

$= 6x^2 - 9x - 4x + 6$ $b = -13$

$= 3x(2x-3) - 2(2x-3)$ $(-9)(-4) = 36$

$= (2x-3)(3x-2)$ $(-9) + (-4) = -13$

b) $10m^2 + m - 3$ $ac = -30$

$= 10m^2 + 6m - 5m - 3$ $b = 1$

$= 2m(5m+3) - 1(5m+3)$ $(6)(-5) = -30$

$= (2m-1)(5m+3)$ $6 + (-5) = 1$

c) $2a^2 - 11a + 12$ $ac = 24$

$= 2a^2 - 8a - 3a + 12$ $b = -11$

$= 2a(a-4) - 3(a-4)$ $(-8)(-3) = 24$

$= (a-4)(2a-3)$ $(-8) + (-3) = -11$

d) $4x^2 - 20x + 25$ $ac = 100$

$= 4x^2 - 10x - 10x + 25$ $b = -20$

$= 2x(2x-5) - 5(2x-5)$ $(-10)(-10) = 100$

$= (2x-5)(2x-5)$ $(-10) + (-10) = -20$

e) $5d^2 - 14d + 8$ $ac = 40$

$= 5d^2 - 10d - 4d + 8$ $b = -14$

$= 5d(d-2) - 4(d-2)$ $(-4)(-10) = 40$

$= (5d-4)(d-2)$ $(-4) + (-10) = -14$

f) $6n^2 + 26n - 20$

$= 2(3n^2 + 13n - 10)$ $ac = -30$

$= 2[3n^2 + 15n - 2n - 10)$ $b = 13$

$= 2[3n(n+5) - 2(n+5)]$ $(15)(-2) = -30$

$= 2(3n-2)(n+5)$ $(15) + (-2) = 13$

7a) $15x^2 + 4x - 4$ $ac = -60$

$= 15x^2 + 10x - 6x - 4$ $b = 4$

$= 5x(3x+2) - 2(3x+2)$ $(10)(-6) = -60$

$= (3x+2)(5x-2)$ $(10) + (-6) = 4$

b) $18m^2 - 3m - 10$ $ac = -180$

$= 18m^2 + 12m - 15m - 10$ $b = -3$

$= 6m(3m+2) - 5(3m+2)$ $(12)(-15) = -180$

$= (6m-5)(3m+2)$ $(12) + (-15) = -3$

c) $16a^2 - 50a + 36$ $ac = 144$

$= 2(8a^2 - 25a + 18)$ $b = -25$

$= 2(8a^2 - 16a - 9a + 18)$ $(-16)(-9) = 144$

$= 2[8a(a-2) - 9(a-2)]$ $(-16) + (-9) = -25$

$= 2(8a-9)(a-2)$

d) $35x^2 - 27x - 18$ $ac = -630$

$= 35x^2 - 42x + 15x - 18$ $b = -27$

$= 7x(5x-6) + 3(5x-6)$ $(-42)(15) = -630$

$= (7x+3)(5x-6)$ $(-42) + (15) = -27$

e) $63n^2 + 126n + 48$

$= 3(21n^2 + 42n + 16)$ $ac = 336$

CANNOT BE FACTORED FURTHER! $b = 42$

f) $24d^2 - 62d + 35$ $ac = 840$

$= 24d^2 - 20d - 42d + 35$ $b = -62$

$= 4d(6d-5) - 7(6d-5)$ $(-20)(-42) = 840$

$= (4d-7)(6d-5)$ $(-20) + (-42) = -62$

8. $(3x-4)(x+1)$ $(3x-4)(x-1)$ $(3x-4)(3x-4)$

$= 3x^2 + 3x - 4x - 4$ $= 3x^2 - 3x - 4x + 4$ $= 9x^2 - 12x - 12x + 16$

$= 3x^2 - x - 4$ $= 3x^2 - 7x + 4$ $= 9x^2 - 24x + 16$

9a) $A = 6x^2 + 17x - 3$ $ac = -18$

$= 6x^2 + 18x - 1x - 3$ $b = 17$

$= 6x(x+3) - 1(x+3)$ $(18)(-1) = -18$

$= (6x-1)(x+3)$ $(18) + (-1) = 17$

l w

b) $A = 8x^2 - 26x + 15$ $ac = 120$ $b = -26$

$= 8x^2 - 20x - 6x + 15$

$= 4x(2x-5) - 3(2x-5)$ $(-20)(-6) = 120$

$= (2x-5)(4x-3)$ $(-20) + (-6) = -26$

l w

10a) kx^2+5x+2

$ac = 2k$

$b = 5$

$(-1)(6) = 2k$ $(2)(3) = 2k$

$(-1)+(6) = 5$ $(2)+(3) = 5$

$2k = -6$ Values of k that

$k = -3$ can work would

$(4)(1) = 2k$ be 3, -3, 2

$(4)+(1) = 5$ Think of pairs of

$2k = 4$ #'s that add to 5

$k = 2$. (there are lots of possibilities!)

b) $9x^2+kx-5$

$ac = -45$

$b = k$

What multiplies to -45?

$1+(-45) = -44$

$-1+45 = 44$

$3+(-15) = -12$

$-3+15 = 12$

$5+(-9) = -4$

$-5+9 = 4$

values of k

11a) $6x^2+34x-12$ $ac = -18$

$= 2(3x^2+17x-6)$ $b = 17$

$= 2[3x^2+18x-1x-6)$ $(18)(-1) = -18$

$= 2[3x(x+6)-1(x+6)]$ $(18)+(-1) = 17$

$= 2(x+6)(3x-1)$

b) $18v^2+33v-30$ $ac = -60$

$= 3(6v^2+11v-10)$ $b = 11$

$= 3(6v^2+15v-4v-10)$ $(15)(-4) = -60$

$= 3[3v(2v+5)-2(2v+5)]$ $(15)+(-4) = 11$

$= 3(3v-2)(2v+5)$

c) $48c^2-160c+100$ $ac = 300$

$= 4(12c^2-40c+25)$ $b = -40$

$= 4[12c^2-30c-10c+25]$ $(-30)(-10) = 300$

$= 4[6c(2c-5)-5(2c-5)]$ $(-30)+(-10) = -40$

$= 4(2c-5)(6c-5)$

d) $5b^3-17b^2+6b$ $ac = 30$

$= b(5b^2-17b+6)$ $b = -17$

$= b(5b^2-15b-2b+6)$ $(-15)(-2) = 30$

$= b[5b(b-3)-2(b-3)]$ $(-15)+(-2) = -17$

$= b(b-3)(5b-2)$

e) $27xy^2-51xy-6x$ $ac = 18$

$= 3x(9y^2-17y-2)$ $b = -17$

$= 3x(9y^2-18y+1y-2)$

$= 3x[9y(y-2)+1(y-2)]$

$= 3x(y-2)(9y+1)$

f) $-7a^2-29a+30$ $ac = -210$

$= -(7a^2+29a-30)$ $b = 29$

$= -(7a^2-6a+35a-30)$ $(-6)(35) = -210$

$= -[a(7a-6)+5(7a-6)]$ $(-6)+(35) = 29$

$= -(7a-6)(a+5)$

Milbury

DOES $k=-5$ SATISFY THE EQUATION?

12a) $k^2+9k-52=0$
$(-5)^2+9(-5)-52 \stackrel{?}{=} 0$
$25-45-52 \stackrel{?}{=} 0$
$-72 \neq 0$ ☹

b) $4(-5)^3+32(-5)^2+60(-5) \stackrel{?}{=} 0$
$300-300=0$
$0=0$ ☺

c) $6(-5)^2+23(-5)+7 \stackrel{?}{=} 0$
$150-115+7 \stackrel{?}{=} 0$
$42 \neq 0$ ☹

d) $10+19(-5)-15(-5)^2 \stackrel{?}{=} 0$
$10-95-375 \stackrel{?}{=} 0$
$-460 \neq 0$ ☹

e) $7(-5)^2+29(-5)-30 \stackrel{?}{=} 0$
$175-145-30 \stackrel{?}{=} 0$
$0=0$ ☺

f) $10(-5)^2+65(-5)+75 \stackrel{?}{=} 0$
$250-325+75 \stackrel{?}{=} 0$
$0=0$ ☺

13a) $y=2x^2-9x+4$ $\quad ac=8$, $b=-9$
$y=2x^2-1x-8x+4$
$y=x(2x-1)-4(2x-1)$
i) $y=(x-4)(2x-1)$

ii) $x=4, x=\frac{1}{2}$

iii) $x=9/4$
$y=2(9/4)^2-9(9/4)+4$
$=-49/8$
$(9/4, -49/8)$

b) $y=-2x^2+7x+15$ $\quad ac=-30$, $b=7$
$y=-2x^2+10x-3x+15$
$=-2x(x-5)-3(x-5)$
$=(-2x-3)(x-5)$
i) $=-(2x+3)(x-5)$
ii) $x=\frac{-3}{2}, x=5$
iii) $x=7/4$
$y=-2(7/4)^2+7(7/4)+15$
$=21\frac{1}{8}$ or $169/8$
$(7/4, 169/8)$

iv)

$ac=36$
$b=20$

14. $P=-4x^2+20x-9$
$=-4x^2+18x+2x-9$
$=-2x(2x-9)+1(2x-9)$
$=(-2x+1)(2x-9)$
$=-(2x-1)(2x-9)$

a) $\frac{1}{2}$ + $4\frac{1}{2}$ "hundred thousand" games.

b) $P=-(2(2.5)-1)(2(2.5)-9)$
$=\$16$ million

c) $x=\frac{\frac{1}{2}+4\frac{1}{2}}{2}$
$=2\frac{1}{2}$ hundred thousand games

15a) $8x^2 - 13xy + 5y^2$ $ac = 40$, $b = -13$, $(-8)(-5) = 40$, $(-8)+(-5) = 13$

$= 8x^2 - 8xy - 5xy + 5y^2$

$= 8x(x-y) - 5y(x-y)$

$= (8x-5y)(x-y)$

b) $5a^2 - 17ab + 6b^2$ $b = -17$, $(-15)(-2) = 30$, $(-15)+(-2) = -17$

$= 5a^2 - 15ab - 2ab + 6b^2$

$= 5a(a-3b) - 2b(a-3b)$

$= (5a-2b)(a-3b)$

c) $-12s^2 - sr + 35r^2$ $ac = -420$, $b = 1$, $(-20)(21) = -420$, $(-20)+(21) = 1$

$= -(12s^2 + sr - 35r^2)$

$= -(12s^2 - 20sr + 21sr - 35r^2)$

$= -[4s(3s-5r) + 7r(3s-5r)]$

$= -(4s+7r)(3s-5r)$

d) $16c^4 + 64c^2 + 39$ $ac = 624$, $b = 64$, $(12)(52) = 624$, $(12)+(52) = 64$

$= 16c^4 + 12c^2 + 52c^2 + 39$

$= 4c^2(4c^2+3) + 13(4c^2+3)$

$= (4c^2+13)(4c^2+3)$

e) $14v^6 - 39v^3 + 27$ $ac = 378$, $b = -39$, $(-18)(-21) = 378$, $(-18)+(-21) = -39$

$= 14v^6 - 18v^3 - 21v^3 + 27$

$= 2v^3(7v^3-9) - 3(7v^3-9)$

$= (7v^3-9)(2v^3-3)$

f) $c^3d^3 + 2c^2d^2 - 8cd$

$= cd(c^2d^2 + 2cd - 8)$ $ac = -30$, $b = -7$, $(-10)(3) = -30$, $(-10)+3 = -7$

17a) $6(a+b)^2 + 11(a+b) + 3$

Let $y = a+b$

$= 6y^2 + 11y + 3$ $ac = 18$, $b = 11$

$= 6y^2 + 9y + 2y + 3$

$= 3y(2y+3) + 1(2y+3)$

$= (3y+1)(2y+3)$

Sub $a+b$ back in!

$= [3(a+b)+1][2(a+b)+3]$

b) $5(x-y)^2 - 7(x-y) - 6$

$= 5(x-y)^2 - 10(x-y) + 3(x-y) - 6$

$= 5(x-y)(x-y-2) + 3(x-y-2)$

$= [5(x-y)+3](x-y-2)$

c) $8(x+1)^2 - 14(x+1) + 3$ $ac = 24$, $b = -14$

$= 8(x+1)^2 - 12(x+1) - 2(x+1) + 3$

$= 4(x+1)(2(x+1)-3) - 1(2(x+1)-3)$

$= (4(x+1)-1)(2(x+1)-3)$

$= (4x+3)(2x-1)$

$ac = -30$, $b = 13$, $(15)(-2) = -30$, $(15)+(-2) = 13$

d) $12(a-2)^4 + 52(a-2)^2 - 40$

$= 4[3(a-2)^4 + 13(a-2)^2 - 10]$

$= 4[3(a-2)^4 + 15(a-2)^2 - 2(a-2)^2 - 10]$

$= 4[3(a-2)^2((a-2)^2+5) - 2((a-2)^2+5)]$

$= 4[(3(a-2)^2 - 2)((a-2)^2+5)]$

$= 4[(3(a^2-4a+4)-2)(a^2-4a+4+5)]$

$= 4[(3a^2-12a+10)(a^2-4a+9)]$

$= 4(3a^2-12a+10)(a^2-4a+9)$

4.5 Factoring Quadratics: Special Cases

1a) $x^2 - 9$

c) $(x+3)(x-3)$

2a) $9x^2 - 4$

$= (3x+2)(3x-2)$

b) $36x^2 + 60x + 25$

$= (6x+5)(6x+5)$

3a) $x^2 - 100$

$= (x+10)(x-10)$

b) $n^2 + 10n + 25$

$= (n+5)(n+5)$

c) $81a^2 - 16$

$= (9a-4)(9a+4)$

d) $20x^2 - 5$

$= 5(4x^2 - 1)$

$= 5(2x+1)(2x-1)$

e) $25m^2 - 70m + 49$

$= (5m-7)^2$

f) $18x^2 - 48x + 32$

$= 2(9x^2 - 24x + 16)$

$= 2(3x-4)^2$

4a) $4x^2 + 20x + 25$

$= (2x+5)^2$

b) $25x^2 - 9$

$= (5x+3)(5x-3)$

c) $16x^2 - 72x + 81$

$= (4x-9)(4x-9)$

d) $9x^2 - 64$

$= (3x-8)(3x+8)$

6a) $x^2 + 10x + 25$

$= (x+5)^2$

b) $b^2 + 8b + 16$

$= (b+4)^2$

c) $m^2 - 4m + 4$

$= (m-2)^2$

d) $4c^2 - 44c + 121$

$= (2c-11)^2$

e) $16p^2 + 72p + 81$

$= (4p+9)^2$

f) $25z^2 - 30z + 9$

$= (5z-3)^2$

7a) $49a^2 + 56a + 16$

$= (7a+4)^2$

b) $4x^2 - 25$

$= (2x+5)(2x-5)$

c) $-50x^2 - 40x - 8$

$= -2(25x^2 + 20x + 4)$

$= -2(5x+2)^2$

d) $4a^2 - 256$

$= 4(a^2 - 64)$

$= 4(a+8)(a-8)$

e) $225 - 16x^2$

$= (15-4x)(15+4x)$

f) $(x+1)^2 + 2(x+1) + 1$

$= [(x+1)+1]^2$

$= (x+2)^2$

8a) $64^2 - 60^2$

$= (64+60)(64-60)$

$= (124)(4)$

$= 496$

b) $18^2 - 12^2$

$= (18+12)(18-12)$

$= (30)(6)$

$= 180.$

9a) $8x^2 - 18x + 9$

is not a perfect square because $\sqrt{8}$ is not an integer.

b) $8x^2 - 18x + 9$

is not a diff. of squares because $b \neq 0$.

10a) $x^4 - 12x^2 + 36$

$= (x^2 - 6)^2$

b) $a^4 - 16$

$= (a^2+4)(a^2-4)$

$= (a^2+4)(a+2)(a-2)$

c) $49x^2 - 100$

$= (7x+10)(7x-10)$

d) $12x^2 - 60x + 75$

$= 3(4x^2 - 20x + 25)$

$= 3(2x-5)^2$

e) $x^4 - 24x^2 + 144$

$= (x^2 - 12)^2$

f) $289x^6 - 81$

$= (17x^3 - 9)(17x^3 + 9)$

11a) $x^2 - 16xy + 64y^2$

$= (x - 8y)^2$

b) $36x^2 - 25y^2$

$= (6x+5y)(6x-5y)$

c) $16x^2 - 72xy + 81y^2$

$= (4x - 9y)^2$

d) $1 - 9a^2b^4$

$= (1+3ab^2)(1-3ab^2)$

e) $-18x^2 + 24xy - 8y^2$

$= -2(9x^2 - 12xy + 4y^2)$

$= -2(3x - 2y)^2$

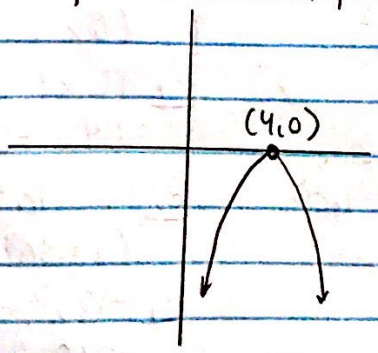
11f) $50x^3 - 8xy^2$
$= 2x(25x^2 - 4y^2)$
$= 2x(5x+2y)(5x-2y)$

12a) $x^2 - c^2 - 8x + 16$
$= x^2 - 8x + 16 - c^2$
$= (x-4)^2 - c^2$
$= (x-4+c)(x-4-c)$

b) I grouped the terms into a perfect square trinomial + factored. That left me with a diff. of squares, so I factored again

b) $4c^2 - a^2 - 6ab - 9b^2$
$= 4c^2 - (a^2 + 6ab + 9b^2)$
$= 4c^2 - (a+3b)^2$
$= (2c + a + 3b)(2c - a - 3b)$
See explanation for a).

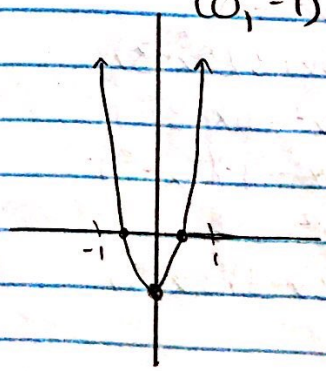
13a) $y = -x^2 + 16x - 64$
$= -(x^2 - 16x + 64)$
i) $= -(x-4)^2$
ii) zero: $(4, 0)$
iii) vertex: $(4, 0)$
opens down, y-int: $(0, -64)$



14a) $A_S = (4x+5)^2 - (x-2)^2$
$= [(4x+5) + (x-2)][(4x+5) - (x-2)]$
$= (3x+3)(3x+7)$
$= 3(x+1)(3x+7)$

b) $A_S = \pi(3x+2)^2 - \pi(x+1)^2$
$= \pi[(3x+2)^2 - (x+1)^2]$
$= \pi[(3x+2) + (x+1)][(3x+2) - (x+1)]$
$= \pi(4x+3)(2x+1)$

b) $y = 4x^2 - 1$
i) $y = (2x+1)(2x-1)$
ii) zeros: $(\frac{1}{2}, 0), (-\frac{1}{2}, 0)$
iii) vertex: $x = 0, y = -1$
$(0, -1)$



4.6 Reasoning About Factoring Polynomials

1a) $6xy + 12x^2y^2 - 4x^3y^3$ - Polynomial,
2a) $2xy(3 + 6xy - 2x^2y^2)$ Common factor

b) $20x^2 + 11x - 3$ → Trinomial,
2b) $= 20x^2 + 15x - 4x - 3$ Decomp
$= 5x(4x+3) - 1(4x+3)$ $ac = -60$
$= (4x+3)(5x-1)$ $b = 11$
$(15)(-4) = -60$
$(15) + (-4) = 11$

1c) $3x^2 + 3xa - 2x - 2a$ → polynomial
2c) $= 3x(x+a) - 2(x+a)$ Grouping
$= (3x-2)(x+a)$

1d) $49y^2 - 9$ → Diff. of Squares,
2d) $= (7y+3)(7y-3)$ Shortcut

1e) $3x^2 - 3x - 90$ → Trinomial,
2e) $= 3(x^2 - x - 30)$ Common, Then
$= 3(x-6)(x+5)$ Decomp

1f) $x^2 - 13x + 42$ → Trinomial,
2f) $= (x-7)(x-6)$ $a=1$, so shortcut

3ai) $x^2 + 2x + 1$
ii) $x^2 - x - 30$
iii) $16x^2 - 9$
iv) $2x^2 + 5x + 3$
v) $3x^2 - 3x - 90$
vi) $x^2 + 3x - 2x - 6$

4a) $-10a^3 + 15a^2$
$= -5a^2(2a+3)$

b) $x^2 - 2x - 63$
$= (x+7)(x-9)$

c) $25x^2 - 49$
$= (5x-7)(5x+7)$

d) $6x^2 + 11x - 10$
$= (2x+5)(3x-2)$

6a) $16x^2 - 25$
$= (4x+5)(4x-5)$

b) $-6b^2a - 9b^3 + 15b^2$
$= -3b^2(2a - 3b - 5)$

c) $c^2 - 12c + 35$
$= (c-7)(c-5)$

d) $49d^2 + 14d + 1$
$= (7d+1)^2$

e) $12x^2 + 4x - 21$ $ac = -252$
$= 12x^2 + 18x - 14x - 21$ $b = 4$
$= 6x(2x+3) - 7(2x+3)$ $(18)(-14) = -252$
$= (6x-7)(2x+3)$ $(18) + (-14) = 4$

f) $2wz + 6w - 5z - 15$
$= 2w(z+3) - 5(z+3)$
$= (z+3)(2w-5)$

7a) $10x^2 + 3x - 1$ $ac = -10$
$= 10x^2 + 5x - 2x - 1$ $b = 3$
$= 5x(2x+1) - 1(2x+1)$ $(5)(-2) = -10$
$= (2x+1)(5x-1)$ $5 - 2 = 3$

b) $144a^4 - 121$
$= (12a^2 + 11)(12a^2 - 11)$

c) $24ac - 8c + 21a - 7$
$= 8c(3a-1) + 7(3a-1)$
$= (8c+7)(3a-1)$

d) $x^3 - 11x^2 + 18x$
$= x(x^2 - 11x + 18)$
$= x(x-9)(x-2)$

e) $18x^2 + 60x + 50$
$= 2(9x^2 + 30x + 25)$
$= 2(3x+5)^2$

f) $x^2y - 4y$
$= y(x^2 - 4)$
$= y(x+2)(x-2)$

21 12 / 42 / 210 / 25 2

5. $3x^2 + 11x + 10$
$= 3x^2 + 6x + 5x + 10$
$= 3x(x+2) + 5(x+2)$
$= (3x+5)(x+2)$

$ac = 30$
$b = 11$
$(6)(5) = 30$
$6 + 5 = 11$

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8a) $2s^2+4s-6$
$=2(s^2+2s-3)$
$=2(s+3)(s-1)$

b) $-w^2-5w+14$
$=-(w^2+5w-14)$
$=-(w+7)(w-2)$

c) z^4-13z^2+36
$=(z^2-9)(z^2-4)$
$=(z+3)(z-3)(z+2)(z-2)$

d) $8s^2-50r^2$
$=2(4s^2-25r^2)$
$=2(2s+5r)(2s-5r)$

e) $49g^2-84g+36$
$=(7g-6)^2$

f) $x^2+10x+25-16y^2$
$=(x+5)^2-16y^2$
$=(x+5+4y)(x+5-4y)$

10a) $xy-ty+xs-ts$
$=y(x-t)+s(x-t)$
$=(x-t)(y+s)$

b) $-25x^2+16y^2$
$=-(25x^2-16y^2)$
$=-(5x+4y)(5x-4y)$

c) a^4-13a^2+36
$=(a^2-9)(a^2-4)$
$=(a+3)(a-3)(a+2)(a-2)$

d) x^2-y^2-2y-1
$=x^2-(y^2+2y+1)$
$=x^2-(y+1)^2$
$=[x+(y+1)][x-(y+1)]$
$=(x+y+1)(x-y-1)$

9a) x^4-1
$=(x^2-1)(x^2+1)$
↑ This is a difference of squares

b) $x^2y-9xy+20y$
$=y(x^2-9x+20)$
This can be factored!

c) $15x^2+6xy-5x-2y$
$=3x(5x+2y)-1(5x+2y)$
we can common factor again!

d) $48a^4c^3-3b^4c^3$
$=3c^3(16a^4-b^4)$
Difference of squares!

10e) $2(a+b)^2+5(a+b)+3$
$=[2(a+b)+3][(a+b)+1]$
$=(2a+2b+3)(a+b+1)$

f) $6x^3-63x-13x^2$
$=x(6x^2-13x-63)$ $ac=-378$
$=x(6x^2+14x-27x-63)$ $b=-13$
$=x[2x(3x+7)-9(3x+7)]$ $(14)(-27)=-378$
$=x(3x+7)(2x-9)$ $14-27=-13$

11. $A=8x^2+18x+7$ $ac=56$
$=8x^2+14x+4x+7$ $b=18$
$=2(4x+7)+1(4x+7)$ $(14)(4)=56$
a) $=(4x+7)(2x+1)$ $14+4=18$
$l \times w$

b) Let $x=3$
$A=(12+7)(6+1)$ $l=19$
$=(19)(7)$ $w=7$
$=133\ \text{unit}^2$

12. $A = \pi x^2 + 10\pi x + 25\pi$

$= \pi(x^2 + 10x + 25)$

$= \pi(x+5)^2$

a) $A = \pi r^2$, so $r = x+5$

b) Let $x = 10$

$r = 15$

$A = \pi(15)^2$

$= 706.9\ cm^2$

13. $V = 2x^3 + 14x^2 + 24x$

$= 2x(x^2 + 7x + 12)$

$= 2x(x+3)(x+4)$

a) $l = 2x$, $w = x+3$, $h = x+4$

b) $V = 2(5)(5+3)(5+4)$

$= 10(8)(9)$ $l = 10, w = 8, h = 9$

$= 720\ cm^3$

14a) $xy + 3x^2y - 4xy + 6x^2y$

$= -3xy + 9x^2y$

$= -3xy(1 - 3x)$

• no, $x+y$ isn't a factor

b) $x^5 - x^3y^2 + x^3 - xy^2$

$= x(x^4 - x^2y^2 + x^2 - y^2)$

$= x(x^2(x^2 - y^2) + 1(x^2 - y^2))$

$= x(x^2+1)(x^2 - y^2)$

$= x(x^2+1)(x+y)(x-y)$

$x+y$ is a factor!

c) $x^2y + 6y - 9xy + x^2y + xy$

$= 2x^2y + 6y - 8xy$

$= 2y(x^2 - 4x + 3)$

$= 2y(x-3)(x-1)$

∴ not a factor

d) $x^3 + 5x^2 + 6x + x^2y + 5xy + 6y$

$= x(x^2 + 5x + 6) + y(x^2 + 5x + 6)$

$= (x+y)(x+2)(x+3)$

$x+y$ is a factor!

16a) $\frac{x^2}{9} - \frac{1}{4}$

$= \left(\frac{x}{3} + \frac{1}{2}\right)\left(\frac{x}{3} - \frac{1}{2}\right)$

b) $100 - (a-5)^2$

$= (10 + (a-5))(10 - (a-5))$

$= (5+a)(15-a)$

c) $4x^2 + y^2$

Can't factor

d) $\frac{25a^2}{64} - \frac{9b^2}{49}$

$= \left(\frac{5a}{8} + \frac{3b}{7}\right)\left(\frac{5a}{8} - \frac{3b}{7}\right)$

e) $(x+3)^2 - (y-3)^2$

$= [(x+3) + (y-3)][(x+3) - (y-3)]$

$= (x+y)(x-y+6)$

f) $4(c-5)^2 + 12(c-5) + 9$

$= (2(c-5) + 3)^2$

$= (2c-7)^2$

17. A-

Chapter 4 Review

1a) $6x - 9$
$= 3(x-3)$

b) $2x^2 + 8x$
$= 2x(x+4)$

2a) $20x^2 - 4x$
$= 4x(5x-1)$

b) $3n^2 - 6n + 15$
$= 3(n^2 - 2n + 5)$

c) $-2x^3 + 6x^2 + 4x$
$= -2x(x^2 - 3x - 2)$

d) $6a(3-7a) - 5(3-7a)$
$= (3-7a)(6a-5)$

3a) $A = 16x^2 - 24$
$= 8(2x^2 - 3)$
length ↗ width ↗

b) Yes, there are other possibilities:
$A = 2(8x^2 - 12)$
length ↗ width ↗

4. $16x^3y + 4x^3y^2$
$= 4x^3y(4+y)$

5a) $x^2 + 3x - 4$
$= (x+4)(x-1)$

b) $2x^2 + 5x - 3$
$= (2x-1)(x+3)$

6a) $x^2 + 16x + 63$
$= (x+9)(x+7)$

b) $x^2 - 7x - 60$
$= (x-12)(x+5)$

c) $x^2 + 6x - 27$
$= (x+9)(x-3)$

d) $5(x^2 - x - 20)$
$= 5(x-5)(x+4)$

7. $y = x^2 + 7x + 12$

a) $y = (x+3)(x+4)$

b) $(-3, 0), (-4, 0)$

c) $x = \frac{-3 + (-4)}{2}$
$= -3\frac{1}{2}$
$y = (-\frac{7}{2})^2 + 7(-\frac{7}{2}) + 12$
$= \frac{49}{4} - \frac{49}{2} + 12$
$= -\frac{49}{4} + 12$
$= -\frac{1}{4}$
$(-3\frac{1}{2}, -\frac{1}{4})$

d) Minimum value: $-\frac{1}{4}$
Occurs at: $x = -3\frac{1}{2}$

8a) $x^2 + 18x + 80$
$= (x+10)(x+8)$

b) $10x^2 - 39x + 14$
$= (5x-2)(2x-7)$

9a) $15x^2 - 4x - 4$
• decomposition

b) $20x^2 + 3x - 2$
• decomposition

c) $7a^2 + 6a - 16$
• decomposition

d) $20y^2 - 17y - 10$
• decomposition

10a) $7x^2 - 19x - 6$
$= 7x^2 - 21x + 2x - 6$
$= 7x(x-3) + 2(x-3)$
$= (7x+2)(x-3)$

b) $4a^2 + 23a + 15$
$= 4a^2 + 20a + 3a + 15$
$= 4a(a+5) + 3(a+5)$
$= (4a+3)(a+5)$

c) $12x^2 - 16x + 5$
$= 12x^2 - 10x - 6x + 5$
$= 2x(6x-5) - 1(6x-5)$
$= (2x-1)(6x-5)$

d) $6n^2 - 11ny - 10y^2$
$= 6n^2 - 15ny + 4ny - 10y^2$
$= 3n(2n-5y) + 2y(2n-5y)$
$= (3n+2y)(2n-5y)$

11. $P = -2n^2 + 120n - 1000$
$= -2(n^2 - 60n + 500)$
$= -2(n-50)(n-10)$

a) They break even at 10 and 50 watches.

b) $n = \frac{10+50}{2}$
$= 30$ watches
$P = -2(30)^2 + 120(30) - 1000$
$= \$800$
∴ Their max profit is $800.

12a) $4x^2 + 12x + 9$
$= (2x + 3)^2$

b) $64x^2 - 9$
$= (8x + 3)(8x - 3)$

13a) $144x^2 - 25$
$= (12x - 5)(12x + 5)$

b) $36a^2 + 12a + 1$
$= (6a + 1)^2$

c) $18x^5 - 512xy^2$
$= 2x(9x^4 - 256y^2)$
$= 2x(3x^2 - 16y)(3x^2 + 16y)$

d) $4(x-2)^2 - 20(x-2) + 25$
$= (2(x-2) - 5)^2$
$= (2x - 9)^2$

e) $(x+5)^2 - y^2$
$= (x + 5 + y)(x + 5 - y)$

f) $x^2 - 6x + 9 - 4y^2$
$= (x-3)^2 - 4y^2$
$= (x - 3 + 2y)(x - 3 - 2y)$

14. $x^2 + 25$ cannot be factored.
$b = 0$, $c = 25$
$ac = 25$ ← two numbers can't add to zero but multiply to positive 25
↳ they need opposite signs to cancel!

15. Opposite operations
• see notes

16a) $7x^2 - 26x - 8$
$= 7x^2 - 28x + 2x - 8$
$= 7x(x-4) + 2(x-4)$
$= (7x + 2)(x - 4)$

b) $64a^6 - 25$
$= (8a^3 - 5)(8a^3 + 5)$

c) $18ac - 12a - 15c + 10$
$= 6a(3c - 2) - 5(3c - 2)$
$= (3c - 2)(6a - 5)$

d) $4x^2y - 44xy + 72y$
$= 4y(x^2 - 11x + 18)$
$= 4y(x - 9)(x - 2)$

e) $20x^2 + 61x + 45$
$= (5x + 9)(4x + 5)$

f) $z^4 - 13z^2 + 40$
$= (z^2 - 8)(z^2 - 5)$

17a) $2s^2 + 3s - 5$
$= (2s + 5)(s - 1)$

b) $-w^2 - 2w + 15$
$= -(w^2 + 2w - 15)$
$= -(w + 5)(w - 3)$

c) $z^4 - 4z^2 - 32$
$= (z^2 - 8)(z^2 + 4)$

e) $25g^2 - 30g + 9$
$= (5g - 3)^2$

d) $16s^2 - 121r^2$
$= (4s + 11r)(4s - 11r)$

f) $x^2 + 16x + 64 - 25y^2$
$= (x + 8)^2 - 25y^2$
$= (x + 8 + 5y)(x + 8 - 5y)$