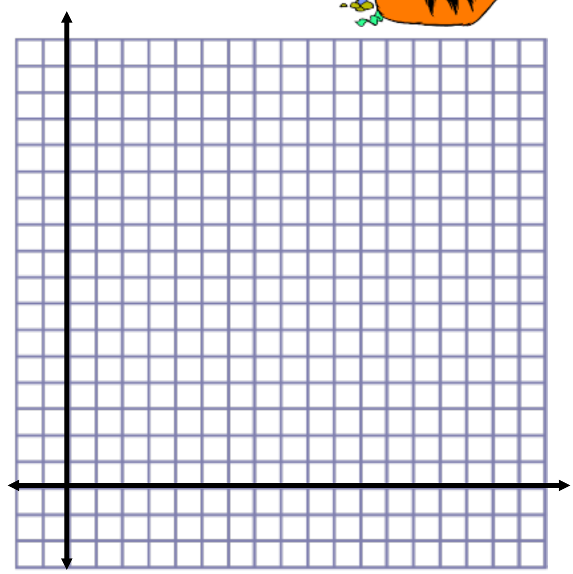
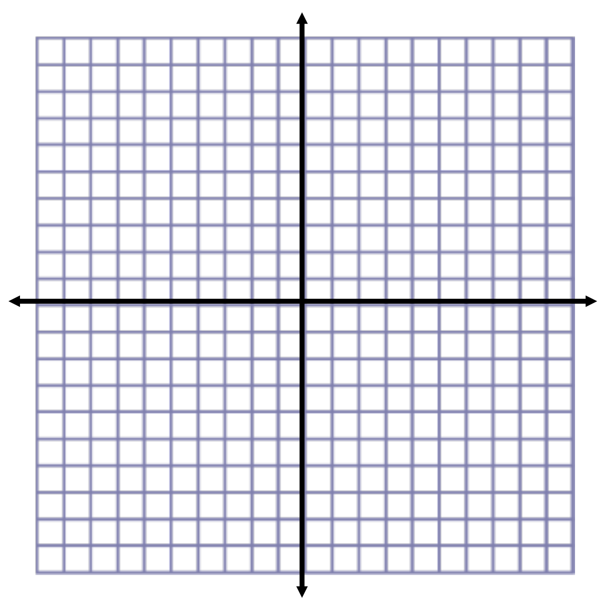
Tuesday, March 3, 2020

2.2 Length of a Line Segment

How can we figure out the exact distance from the origin to the point A(4, 6)?



How can we figure out the exact distance from A(-6, -7) to B(5, 9)? This will be the length of line segment AB.

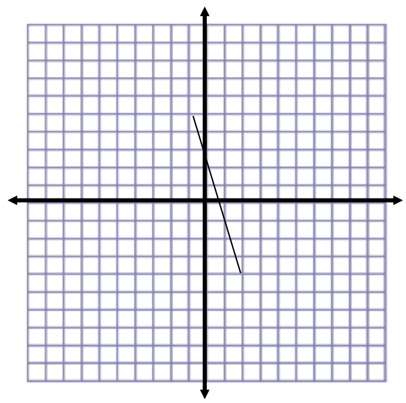


What concept that you spent lots of time on in grade nine was very similar to finding length?

If you wanted to find the length of a line segment without creating an accurate graph, what would the formula look like?

Practice Problem

Find the length of line segment PQ if P(-1, 5) and Q(2, -4). Leave your answer as an **exact value**.



Exact value means that you will leave your answer as a whole number, if it happens to be a perfect square (1, 4, 9, etc.) or a square root. ***Once again, the expectation is that answers will be left in exact form, NOT DECIMALS***. Decimals are okay when dealing with a real life situation where rounding makes sense, but otherwise leave it as a root!

Extending Our Understanding

Some of you have calculators that will simplify roots and give you a "mixed radical".

ex/ Type in $\sqrt{12}$ and press equals. Some of you will get a decimal value, but others will get $2\sqrt{3}$. There are more examples of this below.

$\sqrt{20} = 2\sqrt{5}$ $\sqrt{24} = 2\sqrt{6}$ $\sqrt{28} = 2\sqrt{7}$

What is happening? Why are these expressions equivalent?

Applications of Length (Distance)

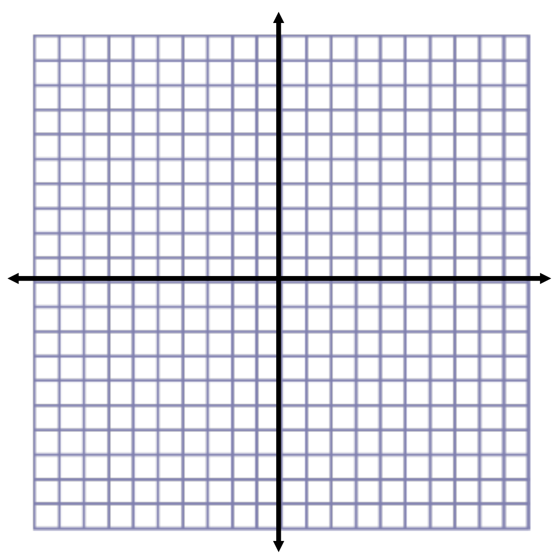
We can use our ability to find distance to help us classify shapes, find perimeters, locate missing vertices of polygons, and find the distance between a point and a line. It is very important that we remember things that we used to know so that we can do this efficiently!

Stuff to Remember:

- calculating slope is often faster than finding distance, and can sometimes take it's place! Also, a picture speaks 1000 words.
- Parallel lines have the same slope.
- Perpendicular lines meet at 90^o and have slopes that are negative reciprocals of each other.
- The points where the sides of polygons meet are called vertices, and a line segment refers to a line that has a distinct starting and ending point.
- **The distance between a point and a line is the shortest distance between them (perpendicular line from the point to the line).**

*When you are asked to show something, or prove something, you need to be sure to do the math that is necessary to actually prove it!

2. A triangle has vertices at A(-1, -1), B(2, 0), and C(1, 3). Show that this is an isosceles right triangle.



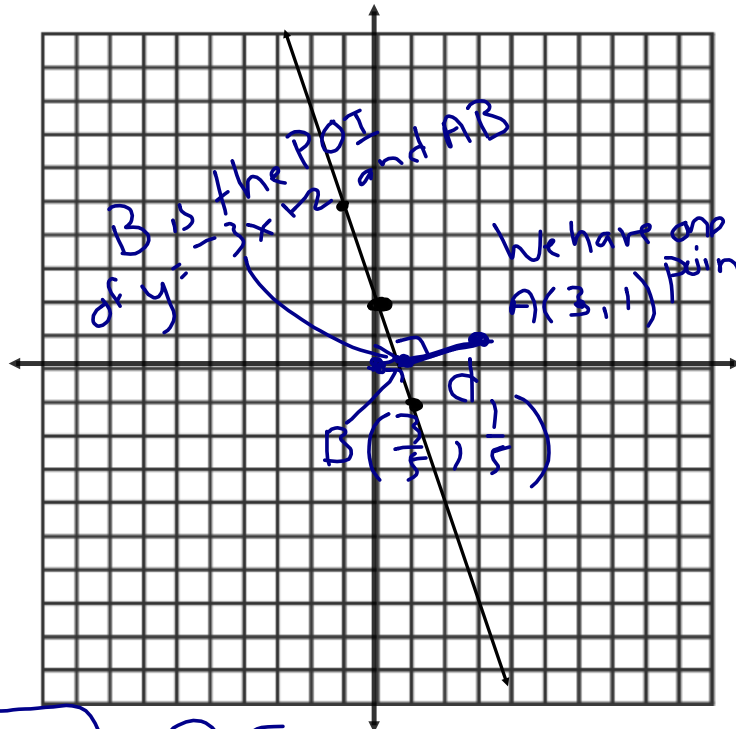
2. Calculate the exact distance between point A(3, 1) and the line y = -3x + 2.

$y = -3x + 2$

slope ↗   ↖ y-int

slope: $-\frac{3}{1}$ $m_{\perp} = \frac{1}{3}$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



① Equation for AB

$A(3,1)$ $m_{AB} = \frac{1}{3}$

$y = mx + b$

$1 = \frac{1}{3}(3) + b$

$1 = 1 + b$

$0 = b$

$\boxed{y = \frac{1}{3}x}$

② Find POI (B)

Sub $y = \frac{1}{3}x$ into $y = -3x + 2$

$3\left(\frac{1}{3}x\right) = 3(-3x) + 3(2)$

$x = -9x + 6$

$\frac{10x}{10} = \frac{6}{10}$

Sub $x = \frac{3}{5}$ into $y = \frac{1}{3}x$

$y = \frac{1}{3}\left(\frac{3}{5}\right)$

$= \frac{3}{15}$ or $\frac{1}{5}$

$B\left(\frac{3}{5}, \frac{1}{5}\right)$

③ Find d_{AB}

$d_{AB}^2 = \left(3 - \frac{3}{5}\right)^2 + \left(1 - \frac{1}{5}\right)^2$

$= \left(\frac{15}{5} - \frac{3}{5}\right)^2 + \left(\frac{5}{5} - \frac{1}{5}\right)^2$

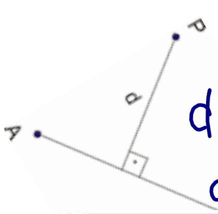
$= \left(\frac{12}{5}\right)^2 + \left(\frac{4}{5}\right)^2$

$d_{AB}^2 = \frac{144}{25} + \frac{16}{25}$

$d_{AB}^2 = \frac{160}{25}$

$d_{AB}^2 = \frac{32}{5}$

$d_{AB} = \sqrt{\frac{32}{5}}$ units



The shortest distance from a point to a line is always the length of the segment that is perpendicular to the line.