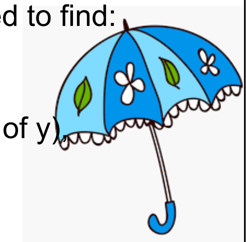
Date: ____________________

6.6 Solving Problems Using Quadratic Models

This section has a mixture of word problems that you can solve using a variety of techniques. It will force you to remember all of the things that we have learned in quadratics so that you can find the most efficient way to solve problems.

Hints:

- If you are given an equation in standard form and are asked to find:
  - > a minimum or maximum,
    - – partial factor (x=-b/2a, sub in to find y)
  - > the roots (can be zeros or the x value at another value of y)
    - – factor if possible
    - – use the quadratic formula

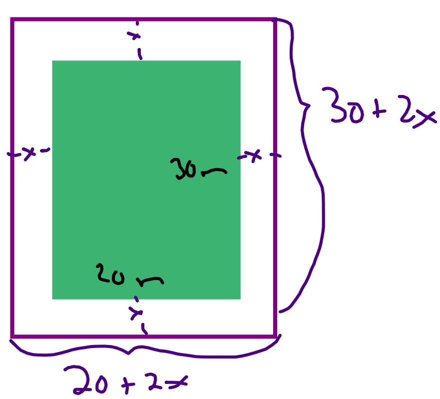


- If you are given an equation in vertex form and are asked to find:
  - > a minimum or maximum
    - – use the equation!!
  - > the roots
    - – rearrange to isolate x (remember to take the + and - square root!!).

Example 1: Tala works at the Botanical Gardens and is planning a new rectangular garden 20 m by 30 m. She plans to build a walkway, with a uniform width, all around the garden. She can afford 600 m^2 of materials to build the walkway. Develop and use a quadratic model to determine the width of the walkway.

$\boxed{\begin{aligned} A_g &= 20 \times 30 \\ &= 600\ m^2 \end{aligned}}$

Let x be the width of the walkway.



Total Area:

$(30+2x)(20+2x) = 1200$

$600 + 60x + 40x + 4x^2 = 1200$

$\frac{4x^2 + 100x - 600}{4} = \frac{0}{4}$

$A_w = 600\ m^2$

$x^2 + 25x - 150 = 0$

$(x+30)(x-5) = 0$

$A_{Total} = A_G + A_W$
$= 1200\ m^2$

$x = -30$

$x = 5$

∴ The walkway is 5 m wide.

Revisiting Revenue Problems - Maximizing Revenue

> revenue - the amount of money that a company brings in
>
> Revenue = price x number of items sold

When you are solving these problems, you need to:

- let x represent the number of price increases/decreases.
- write an expression to represent the change in price
  (initial price + (increase amount)x)
- write an expression to represent the change in sales
  (initial # of sales - (decrease in sales)x)
- multiply them together.
- find the vertex.

Let's try one! These types of problems will come up all of the way to calculus, so you may as well learn to deal with them now!

Example 2: Jaxson has opened a sandwich shop and sells cheesesteak sandwiches for $8 each. At this price, he usually sells 100 sandwiches per day. He has figured out that for every $0.50 increase in the price, he will sell five fewer sandwiches.

a) Use a quadratic model to determine the maximum revenue that Jaxson can earn and the optimal price for one sandwich.

∴ The price is $9 & the max revenue is $810.

Let x be the # of $0.50 price increases.

$R = (8 + 0.5x)(100 - 5x)$

A.o.S $x = \frac{20 - 16}{2}$

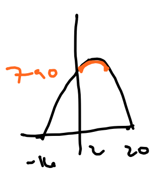
$x = 2$

$8 + 0.5x = 0$
$0.5x = -8$
$x = -16$

$100 - 5x = 0$
$5x = 100$
$x = 20$

$R = (8 + 0.5(2))(100 - 5(2))$
$= (9)(90)$
$= \$810$

b) If Jaxson needs to earn ~~$890~~ 790 per day to stay open, how many sandwiches does he need to sell? (This will be a RANGE of values!)



$(8 + 0.5x)(100 - 5x) = 790$

$800 - 40x + 50x - 2.5x^2 = 790$

$-2.5x^2 + 10x + 10 = 0$

$x = \frac{-10 \pm \sqrt{(10)^2 - 4(-2.5)(10)}}{2(-2.5)}$

$= \frac{-10 \pm \sqrt{200}}{-5}$

$x = \frac{-10 + \sqrt{200}}{-5}$
$= -0.8$

$x = \frac{-10 - \sqrt{200}}{-5}$
$= 4.8$

# of sandwiches

(1) $100 - 5(-0.8) = 104$ sandwiches

(2) $100 - 5(4.8) = 76$ sandwiches

∴ Jaxson has to sell between 76 & 104 sandwiches to stay open.