6.1 Solving Quadratic Equations

1a) $x^2 - 4x + 4 = 0$
$(x-2)^2 = 0$
$x = 2$

b) $2x^2 - 9x - 5 = 0$
$2x^2 - 10x + x - 5 = 0$
$2x(x-5) + 1(x-5) = 0$
$(2x+1)(x-5) = 0$
$x = -\frac{1}{2}$ $x = 5$

2a) $(-3, 0)$, $(3, 0)$

b) $(-\frac{1}{2}, 0)$, $(3, 0)$

3a) $x(x+4) = 0$
$x = 0$, $x = -4$

b) $(x+10)(x+8) = 0$
$x = -10$ $x = -8$

c) $(x-5)^2 = 0$
$x = 5$

d) $(3x+8)(x-4) = 0$
$x = -8/3$, $x = 4$

e) $x^2 + 5x + 6 = 0$
$(x+3)(x+2) = 0$
$x = -3$ $x = -2$

f) $x^2 - 2x - 8 = 0$
$(x-4)(x+2) = 0$
$x = 4$, $x = -2$

4a) $x^2 + x - 6 = 0$
Let $x = 2$
$LS = (2)^2 + 2 - 6$
$= 4 + 2 - 6$
$= 0$
$\therefore x = 2$ is a root for the equation

4b) $x^2 + 7x - 8 = 0$
Let $x = 4$
$LS = (4)^2 + 7(4) - 8$
$= 16 + 28 - 8$
$= 36$
$\therefore x = 4$ is not a root ($LS \neq RS$)

c) $2x^2 + 11x + 5 = 0$
Let $x = -\frac{1}{2}$
$LS = 2(-\frac{1}{2})^2 + 11(-\frac{1}{2}) + 5$
$= 2(\frac{1}{4}) - \frac{11}{2} + 5$
$= \frac{1}{2} - \frac{11}{2} + 5$
$= -5 + 5$
$= 0$
$\therefore x = -\frac{1}{2}$ is a root.

d) $8x^2 + 10x - 3 = 0$
Let $x = \frac{3}{2}$
$LS = 8(\frac{3}{2})^2 + 10(\frac{3}{2}) - 3$
$= 8(\frac{9}{4}) + 15 - 3$
$= 18 + 15 - 3$
$= 30$
$\therefore x = \frac{3}{2}$ is not a root ($LS \neq RS$)

e) $x^2 - 4x - 5 = 0$
Let $x = -5$
$LS = (-5)^2 - 4(-5) - 5$
$= 25 + 20 - 5$
$= 40$
$\therefore x = -5$ is not a root ($LS \neq RS$)

f) $3x^2 - 2x - 8 = 0$
Let $x = 2$
$LS = 3(2)^2 - 2(2) - 8$
$= 12 - 4 - 8$
$= 0$
$\therefore x = 2$ is a root

5a) $x^2 + 2x - 15 = 0$
$(x+5)(x-3) = 0$
$x = -5$, $x = 3$

b) $x^2 + 5x - 24 = 0$
$(x+8)(x-3) = 0$
$x = -8$ $x = 3$

c) $x^2 + 4x + 4 = 0$
$(x+2)^2 = 0$
$x = -2$

d) $x^2 - 5x = 0$
$x(x-5) = 0$
$x = 0$, $x = 5$

e) $x^2 - 6x - 16 = 0$
$(x-8)(x+2) = 0$
$x = 8$, $x = -2$

f) $x^2 - 7x + 12 = 0$
$(x-4)(x-3) = 0$
$x = 4$, $x = 3$

6a) $3x^2 - 5x - 2 = 0$
$(3x+1)(x-2) = 0$
$x = -\frac{1}{3}$, $x = 2$

b) $2x^2 + 3x - 2 = 0$
$(2x-1)(x+2) = 0$
$x = \frac{1}{2}$, $x = -2$

c) $3x^2 - 4x - 15 = 0$
$(3x+5)(x-3) = 0$
$x = -\frac{5}{3}$, $x = 3$

6d) $6x^2 - x - 2 = 0$

$(3x-2)(2x+1) = 0$

$x = \frac{2}{3}, x = -\frac{1}{2}$

e) $4x^2 - 4x - 3 = 0$

$(2x+1)(2x-3) = 0$

$x = -\frac{1}{2}, x = \frac{3}{2}$

f) $9x^2 - 6x + 1 = 0$

$(3x-1)^2 = 0$

$x = \frac{1}{3}$

7a) $x(x+1) = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4, x = 3$

b) $2x(x+4) = x+4$

$2x^2 + 8x - x - 4 = 0$

$2x^2 + 7x - 4 = 0$

$(2x-1)(x+4) = 0$

$x = \frac{1}{2}, x = -4$

c) $3x(x+2) = 2x^2 - (4-x)$

$3x^2 + 6x - 2x^2 = -4 + x$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x = -1, x = -4$

d) $3x(x+6) + 50 = 2x^2 + 3(x-2)$

$3x^2 + 18x + 50 = 2x^2 + 3x - 6$

$x^2 + 15x + 56 = 0$

$(x+8)(x+7) = 0$

$x = -8, x = -7$

e) $(x+2)^2 + x = 2(3x+5)$

$x^2 + 4x + 4 + x = 6x + 10$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, x = -2$

7f) $(2x+1)^2 = x+2$

$4x^2 + 4x + 1 = x + 2$

$4x^2 + 3x - 1 = 0$

$(4x-1)(x+1) = 0$

$x = \frac{1}{4}, x = -1$

8a) $x^2 + 4x - 32 = 0$

$(x+8)(x-4) = 0$

$x = -8, x = 4$

b) $x^2 + 11x + 30 = 0$

$(x+6)(x+5) = 0$

$x = -6, x = -5$

c) $5x^2 - 28x - 12 = 0$

$(5x+2)(x-6) = 0$

$x = -\frac{2}{5}, x = 6$

d) $x^2 + 5x - 14 = 0$

$(x+7)(x-2) = 0$

$x = -7, x = 2$

e) $4x^2 - 20x + 25 = 0$

$(2x-5)^2 = 0$

$x = 5/2$

f) $3x^2 + 16x - 12 = 0$

$(3x-2)(x+6) = 0$

$x = 2/3, x = -6$

9a) $x^2 + 5x - 2 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-2)}}{2}$

$x = 0.37 \quad x = -5.37$

b) $4x^2 - 8x + 3 = 0$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(3)}}{8}$

$= \frac{8 \pm 4}{8}$

$x = 1.5 \quad x = 0.5$

9c) $3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

d) $x(x-5) = 2x+7$

$x^2 - 7x - 7 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(-7)}}{2}$

$x = 7.89,\ x = -0.88$

e) $3x^2 + 5x - 3 = x^2 + 4x + 1$

$2x^2 + x - 4 = 0$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-4)}}{4}$

$x = 1.19 \quad x = 1.69$

f) $(x+3)^2 - 2x = 15$

$x^2 + 6x + 9 - 2x - 15 = 0$

$x^2 + 4x - 6 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-6)}}{2}$

$x = 1.16,\ x = -5.16$

10. $P = -5x^2 + 200x - 1500$

a) Let $P = 0$

$-5(x^2 - 40x + 300) = 0$

$(x-30)(x-10) = 0$

$x = 30,\ x = 10$

∴ He needs to charge $10 or $30 per lawn.

b) Let $P = 500$

$500 = -5x^2 + 200x - 1500$

$-5x^2 + 200x - 2000 = 0$

$-5(x^2 - 40x + 400) = 0$

$-5(x-20)^2 = 0$

$x = \$20$

He needs to charge $20.

11. $A = 4x^2 + 160x$

Let $A = 900$.

$4x^2 + 160x - 900 = 0$

$x^2 + 40x - 225 = 0$

$(x-5)(x+45) = 0$

∴ The width is 5m.

12. $E = \frac{1}{50} R(1650 - R)$

a) $E = \frac{1}{50}(900)(1650-900)$

$= \frac{1}{50}(900)(750)$

$= \$13500$ ∴ He earns $13500

b) $13000 = \frac{1}{50} R(1650 - R)$

$13000 = 33R - \frac{1}{50} R^2$

$\frac{1}{50} R^2 - 33R + 13000 = 0$

$R^2 - 1650R + 650000 = 0$

$(R - 650)(R - 1000) = 0$

∴ It needs to charge between $650 and $1000 rent.

13. ① $y = -2x + 7$, ② $y = 2x^2 + 3x - 5$

Sub ① in to ②.

$-2x + 7 = 2x^2 + 3x - 5$

$2x^2 + 5x - 12 = 0$

$(2x - 3)(x + 4) = 0$

$x = 3/2,\ x = -4$

$y = -2(\frac{3}{2}) + 7$ $\quad y = -2(-4) + 7$

$= 4$ $\quad = 15$

∴ The POIs are $(\frac{3}{2}, 4)$ & $(-4, 15)$

14. $h = -5t^2 - 4t + 120$

a) $-5t^2 - 4t + 120 = 0$

$t = \frac{4 \pm \sqrt{(-4)^2 - 4(-5)(120)}}{-10}$

$t = -5.3,\ t = 4.5$

∴ It will land after 4.5s.

b) $95 = -5t^2 - 4t + 120$

$5t^2 + 4t - 25 = 0$

$t = \frac{-4 \pm \sqrt{(4)^2 - 4(5)(-25)}}{10}$

$t = 1.87 \quad t = -2.67$

∴ It is above 95m for 1.87s.

15. Yes → graph it, rearrange from vertex form, or use the quadratic formula (6.4).

16a) It has to be of the form $ax^2+bx+c=0$ to solve.
b) Let $y=0$.

17. $x^4-9x^2+20=0$ $\quad x^3-9x^2+20x=0$

$x^2-9x+20=0$ $\quad x(x^2-9x+20)=0$
$(x-5)(x-4)=0$ $\quad x(x-5)(x-4)=0$
$x=5, x=4$ $\quad x=0, x=5, x=4$

$x^4-9x^2+20=0$
$(x^2-5)(x^2-4)=0$
$x^2=5$ $\quad x^2=4$
$x=\pm\sqrt{5}$ $\quad x=\pm 2$

18. No!

6.4 The Quadratic Formula

1a) $a=1, b=5, c=-2$

b) $a=4, b=0, c=3$

c) $a=1, b=6, c=0$

d) $2x^2-10x-x^2-1=0$

$x^2-10x-1=0$

$a=1, b=-10, c=-1$

2ai) $x^2+18x-63=0$

$(x+21)(x-3)=0$

$x=-21, x=3$

ii) $x=\frac{-18\pm\sqrt{(18)^2-4(1)(-63)}}{2}$

$x=3, \quad x=-21$

iii) Factoring is shorter in this case.

bi) $8x^2-10x-3=0$

$(4x+1)(2x-3)=0$

$x=-\frac{1}{4}, x=\frac{3}{2}$

ii) $x=\frac{10\pm\sqrt{(10)^2-4(8)(-3)}}{2(8)}$

$x=\frac{3}{2}, \quad x=-\frac{1}{4}$

iii) Factoring is still more efficient!

3a) $\frac{2x^2}{2}=\frac{50}{2}$

$\sqrt{x^2}=\sqrt{25}$

$x=\pm 5$

b) $x^2-1=0$

$x^2=1$

$x=\pm 1$

c) $3x^2-2=10$

$3x^2=12$

$x^2=4$

$x=\pm 2$

d) $x^2-2x-36+2x=0$

$x^2-36=0$

$(x+6)(x-6)=0$

$x=\pm 6$

6. Yes, all of the equations in #5 are factorable, so I could have done that.

7. You would find roots that are whole #'s or fractions.

4a) $(x+1)^2-16=0$

$[x+1+4][x+1-4]=0$

$(x+5)(x-3)=0$

$x=-5, x=3$

b) $-2(x+5)^2+2=0$

$-2(x+5)^2=-2$

$(x+5)^2=1$

$x+5=\pm 1$

$x=1-5, \quad x=-1-5$

$x=-4, \quad x=-6$

c) $-3(x-7)^2+3=0$

$-3(x-7)^2=-3$

$(x-7)^2=1$

$x-7=\pm 1$

$x=1+7 \quad x=-1+7$

$x=8 \quad x=6$

d) $4(x-2)^2-5=0$

$4(x-2)^2=5$

$(x-2)^2=\frac{5}{4}$

$x-2=\pm\sqrt{\frac{5}{4}}$

$x=2+\sqrt{\frac{5}{4}}, x=2-\sqrt{\frac{5}{4}}$

$=3.12 \quad =0.88$

e) $-6(x+3)^2+12=0$

$-6(x+3)^2=-12$

$(x+3)^2=2$

$x+3=\pm\sqrt{2}$

$x=-3+\sqrt{2} \quad x=-3-\sqrt{2}$

$x=-1.59 \quad x=-4.41$

f) $0.25(x-4)^2-4=0$

$(x-4)^2=16$

$x-4=\pm 4$

$x=4+4 \quad x=-4+4$

$=8 \quad =0$

5a) $x = \frac{1 \pm \sqrt{(-1)^2 - 4(6)(-15)}}{2(6)}$

$= \frac{1 \pm \sqrt{361}}{12}$

$x = \frac{5}{3} \quad x = -\frac{3}{2}$

b) $x = \frac{20 \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)}$

$x = \frac{20}{8}$

$x = 5/2$

c) $x = \frac{0 \pm \sqrt{(0)^2 - 4(1)(-16)}}{2(1)}$

$= \frac{\pm\sqrt{64}}{2}$

$x = \pm 4$

d) $x = \frac{-11 \pm \sqrt{(-11)^2 - 4(5)(6)}}{2(5)}$

$x = \frac{-11 \pm \sqrt{121}}{10}$

$x = 0, x = -\frac{11}{5}$

e) $x = \frac{-9 \pm \sqrt{(9)^2 - 4(1)(20)}}{2(1)}$

$= \frac{-9 \pm 1}{2}$

$x = -4, \quad x = -5$

f) $x = \frac{17 \pm \sqrt{(17)^2 - 4(12)(-40)}}{2(12)}$

$x = \frac{17 \pm \sqrt{2209}}{24}$

$x = \frac{8}{3}, \quad x = -5/4$

8a) $x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{8}}{2}$

$x = 3.41, \quad x = 0.59$

b) $x = \frac{6 \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)}$

$= \frac{6 \pm \sqrt{76}}{10}$

$x = 1.47 \quad x = -0.27$

c) $w = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$

$= \frac{-8 \pm \sqrt{40}}{6}$

$w = -0.28 \quad w = -2.39$

d) $2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

$x = \frac{3}{2} \quad x = -1$

e) $m = \frac{5 \pm \sqrt{(5)^2 - 4(1)(3)}}{2(1)}$

$= \frac{5 \pm \sqrt{37}}{2}$

$m = 5.54 \quad m = -0.54$

f) $x = \frac{-12 \pm \sqrt{(12)^2 - 4(-3)(1.7)}}{2(-3)}$

$= \frac{-12 \pm \sqrt{60}}{-6}$

$x = 3.29 \quad x = 0.71$

9a) $2x^2 - 5x = 3x + 12$

$2x^2 - 8x - 12 = 0$

$2(x^2 - 4x - 6) = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2}$

$x = \frac{4 \pm \sqrt{40}}{2}$

$x = 5.16 \quad x = -1.16$

b) $(x+4)^2 = 2(x+5)$

$x^2 + 8x + 16 = 2x + 10$

$x^2 + 6x + 6 = 0$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2(1)}$

$= \frac{-6 \pm \sqrt{12}}{2}$

$x = -1.27 \quad x = -4.73$

c) $x(x+3) = 5 - x^2$

$x^2 + 3x = 5 - x^2$

$2x^2 + 3x - 5 = 0$

$(2x+5)(x-1) = 0$

$x = -5/2, \quad x = 1$

d) $3x(x+4) = (4x-1)^2$

$3x^2 + 12x = 16x^2 - 8x + 1$

$13x^2 - 20x + 1 = 0$

$x = \frac{20 \pm \sqrt{(-20)^2 - 4(13)(1)}}{2(13)}$

$= \frac{20 \pm \sqrt{348}}{26}$

$x = 1.49 \quad x = 0.052$

e) $(x-2)(2x+3) = x + 1$

$2x^2 + 3x - 4x - 6 = x + 1$

$2x^2 - 2x - 7 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-7)}}{2(2)}$

$= \frac{2 \pm \sqrt{60}}{4}$

$x = 2.44 \quad x = -1.44$

f) $(x-3)^2 + 5 = 3(x+1)$

$x^2 - 6x + 9 + 5 = 3x + 3$

$x^2 - 9x + 11 = 0$

$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(11)}}{2(1)}$

$x = \frac{9 \pm \sqrt{37}}{2}$

$x = 7.54 \quad x = 1.46$

10a) $2x^2 + 5x - 14 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-14)}}{2(2)}$

$= \frac{-5 \pm \sqrt{137}}{4}$

$x = 1.67 \quad x = -4.17$

b) $3x^2 + 7.5x - 21 = 0$

$x = \frac{-7.5 \pm \sqrt{(7.5)^2 - 4(3)(-21)}}{2(3)}$

$= \frac{-7.5 \pm \sqrt{308.25}}{6}$

$x = 1.68 \quad x = -4.18$

c) $3x(0.4x+1) = 8.4$

$1.2x^2 + 3x - 8.4 = 0$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1.2)(-8.4)}}{2(1.2)}$

$= \frac{-3 \pm \sqrt{49.32}}{2.4}$

$x = 1.68 \quad x = -4.18$

d) $0.2x^2 + 0.5x - 1.4 = 0$

$x = \frac{-0.5 \pm \sqrt{(0.5)^2 - 4(0.2)(-1.4)}}{2(0.2)}$

$= \frac{-0.5 \pm \sqrt{1.37}}{0.4}$

$x = 1.68 \quad x = -4.18$

11a) They are all the same.

b) The equations are all multiples of each other.

12. ① $y = 2x^2 + 5x - 8$

② $y = -3x^2 + 8x - 1$

* Remember from Ch. 1, POI means $y = y$! Sub ① in to ②.

$2x^2 + 5x - 8 = -3x^2 + 8x - 1$

$5x^2 - 3x - 7 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(-7)}}{2(5)}$

$x = \frac{3 \pm \sqrt{149}}{10}$

$x = 1.52 \quad x = -0.92$

* To find points, sub x back in to ①.

$y = 2(1.52)^2 + 5(1.52) - 8$

$= 4.22 \qquad (1.52, 4.22)$

$y = 2(-0.92)^2 + 5(-0.92) - 8$

$= -10.91 \qquad (-0.92, -10.91)$

13. $x^2 + (x+7)^2 = (x+9)^2$

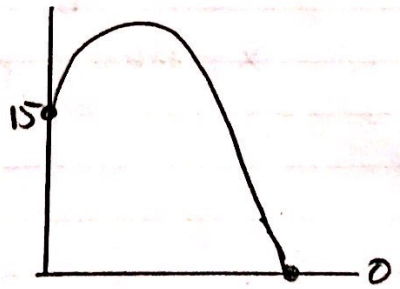
$x^2 + x^2 + 14x + 49 = x^2 + 18x + 81$

$x^2 - 4x - 32 = 0$

$(x-8)(x+4) = 0$

$x = 8, x = -4$

∴ $x = 8$!



14. $h = -4.9t^2 + 1.2t + 15$

a) $-4.9t^2 + 1.2t + 15 = 15$

$-4.9t^2 + 1.2t = 0$

$-t(4.9t - 1.2) = 0$

$t = 0 \qquad t = 0.245\text{s}$

∴ It is above 15m for 0.245s.

b) $-4.9t^2 + 1.2t + 15 = 0$

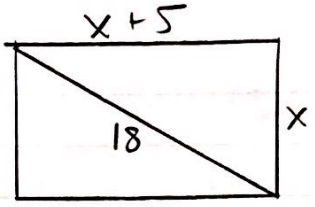
$t = \frac{-1.2 \pm \sqrt{(1.2)^2 - 4(-4.9)(15)}}{2(-4.9)}$

$= \frac{-1.2 \pm \sqrt{295.44}}{-9.8}$

$t = -1.63 \quad t = 1.88\text{s}$

∴ The diver is in the air for 1.88s.

15.



$x^2 + (x+5)^2 = 18^2$

$x^2 + x^2 + 10x + 25 = 324$

$2x^2 + 10x - 299 = 0$

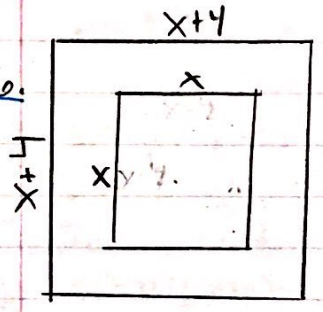
$x = \frac{-10 \pm \sqrt{(10)^2 - 4(2)(-299)}}{2(2)}$

$= \frac{-10 \pm \sqrt{2492}}{4}$

$x = 9.98 \quad x = -14.98$

∴ The rectangle is approximately 9.98cm by 14.98cm.

16.



Let x be the dimensions of the lawn.

$A_{lawn} = A_{walkway}$

$x^2 = 8x + 16$

$x^2 - 8x - 16 = 0$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-16)}}{2}$

$= \frac{8 \pm \sqrt{128}}{2}$

$x = 9.7m \quad x = -1.7m$?

$A_{walkway} = (x+4)^2 - x^2$

$= x^2 + 8x + 16 - x^2$

$= 8x + 16$

∴ The lawn is 9.7m × 9.7m.

18. ① $y = 2x + 5$

② $x^2 + y^2 = 36$

Sub ① in to ②

$x^2 + (2x+5)^2 = 36$

$x^2 + 4x^2 + 20x + 25 = 36$

$5x^2 + 20x - 11 = 0$

$x = \frac{-20 \pm \sqrt{(20)^2 - 4(5)(-11)}}{2(5)}$

$= \frac{-20 \pm \sqrt{620}}{10}$

$x = 0.49 \quad x = -4.49$

Sub back in to ①:

$y = 2(0.49) + 5$

$= 5.98 \quad (0.49, 5.98)$

$y = 2(-4.49) + 5$

$= -3.98 \quad (-4.49, -3.98)$

19a) $(x+3)(x-5)=0$

$x^2-5x+3x-15=0$

$x^2-2x-15=0$

b) $x=\frac{2\pm\sqrt{2^2-4(\frac{3}{2})(\frac{1}{6})}}{3}$ $\quad b=2$, $a=\frac{3}{2}$, $c=-\frac{1}{6}$

$\therefore \frac{3}{2}x^2-2x-\frac{1}{6}=0$

$\times 6 \quad 9x^2-12x-1=0$

20. 3, 4, 5 (Pyth. Triple)

6.5 Interpreting Quadratic Equation Roots

1a) $x^2 - 6x + 5 = 0$

$(x-3)(x-2) = 0$

$x = 3, x = 2$

$x = \frac{6 \pm \sqrt{(6)^2 - 4(1)(5)}}{2(1)}$

$= \frac{6 \pm 4}{2}$

$x = 3, x = 2$

b) The graph has zeros when $x = 3$ and $x = 2$.

c) $b^2 - 4ac$

$= (-6)^2 - 4(1)(5)$

$= 16$

Positive, so 2 real roots.

2a) none → a+k have the same sign

b) two → a+k have opposite sign

c) one → $k = 0$

3a) $(3)^2 - 4(1)(-5)$

$= 29$

∴ Two real roots

b) $(5)^2 - 4(6)(12)$

$= -263$

∴ No real roots

c) $(8)^2 - 4(-1)(-12)$

$= 16$

∴ Two real roots

d) $(8)^2 - 4(-2)(-8)$

$= 0$

∴ one real root

e) $3x^2 - 3x - 12 = 0$

$(-3)^2 - 4(3)(-12)$

$= 153$

∴ Two real roots

f) $4x^2 + 12x + 9 = 0$

$(12)^2 - 4(4)(9)$

$= 0$

∴ One real root.

4a) Twice → a+k have opposite signs

b) None → a+k have the same sign

c) One → $k = 0$

d) None → a+k have the same signs

e) Two → opens down, goes up

f) One → vertex is (0, 0)

5a) None → a+k have the same sign

b) $(3)^2 - 4(-2)(-7)$

$= -47$

∴ No real roots

c) $(-7)^2 - 4(1)(0)$

$= 49$

∴ Two real roots

d) One zero at $x = -2$

e) $(6)^2 - 4(3)(-8)$

$= 132$

∴ Two real roots

f) Two, a+k have opposite signs

6. $r = -2n^2 + 5n + 250$

$500 = -2n^2 + 5n + 250$

$2n^2 - 5n + 250 = 0$

$b^2 - 4ac$

$= (-5)^2 - 4(2)(250)$

$= -1975$

∴ It is impossible for her to earn $500 revenue.

7. $h = 0.005x^2 + 24$

a) No zeros → opens up, moved up

b) 24 m.

8. $h = -4.9t^2 + 10.78t + 1.071$

b) $(10.78)^2 - 4(-4.9)(1.071)$
$= 137.2$
∴ It has two zeros

a) Two → it has to land

d) Find max height.
$t = \frac{-10.78}{-9.8}$
$= 1.1$
$h = -4.9(1.1)^2 + 10.78(1.1) + 24$
≈ 8.329

c) It will pass through 8.7m twice, but 9m never!

9a) $(-4)^2 - 4(2)(1.5)$
$= 4$
∴ Two real roots, so vertex is below (opens up)

b) $(-2)^2 - 4(0.5)(0.5)$
$= 3$
∴ Two real roots, so vertex is above (opens down)

c) $(-30)^2 - 4(5)(45)$
$= -1125$
∴ No real roots, lies above (opens up)

d) $(-4)^2 - 4(0.5)(7.75)$
$= 0.5$
∴ Two real roots. Lie below (opens up)

10. $h = 5x^2 + 6x + k$
$x = \frac{-6}{10}$
$= -\frac{3}{5}$
Let $h = 0$
$5(-\frac{3}{5})^2 + 6(-\frac{3}{5}) + k = 0$
$k = \frac{9}{5}$

b) One zero when $k = \frac{9}{5}$

a) Two zeros when $k < \frac{9}{5}$

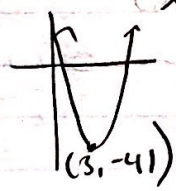
c) No real roots when $k > \frac{9}{5}$

11. $h = -5t^2 + t + 216$

a) 216 m

b) $-5t^2 + t + 216 = 0$
$(1)^2 - 4(-5)(216)$
$= 4321$ ∴ Two solutions, The positive one makes sense!

12. $y = 4x^2 + 24x - 5$
Find the minimum.
$x = \frac{-24}{8}$
$= -3$
$y = 4(-3)^2 + 24(-3) - 5$
$= -41$



(-3, -41) ∴ $y < -41$ will result in no real roots.

13. $x^2 + (x+k)^2 = 25$
$x^2 + x^2 + 2kx + k^2 = 25$
$2x^2 + 2kx + k^2 - 25 = 0$
$(2k)^2 - 4(2)(k^2 - 25) = 0$
$4k^2 - 8k^2 + 200 = 0$
$-4k^2 = -200$
$k^2 = 50$
$k = \pm\sqrt{50}$

14. Agree → factored form only works if it has real roots

17. $y = (x+3)^2$ ① $y = -2x^2 - 5$ ②

Sub ① in to ②

$-2x^2 - 5 = (x+3)^2$

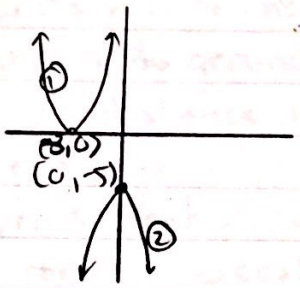
$-2x^2 - 5 = x^2 + 6x + 9$

$-3x^2 - 6x - 14 = 0$

$(-6)^2 - 4(-3)(-14)$

$= -132$

∴ They never intersect (see diagram)



6.6 Solving Problems Using Quadratic Models

1a) zeros → when $h=0$, so when the ball hits the ground (and when it left the ground)
Vertex → when the ball reaches its max height (x), and the max height (y)

b) zeros → when $h=0$, so when it hits the ground, and one Θ value
vertex → max height (y) and the distance it was at when it got there (x)

c) zeros → selling prices that produce 0 profit (break even)
vertex → max profit + the price that produces it.

d) zeros → # of items produced to result in 0 cost
vertex → # of items produced to minimize cost

e) no zeros → we don't want the swing to hit the ground!
vertex → min. height above the ground + the time it occurs.

2. $h = -5t^2 + 22t + 15$

a) The school is 15 m tall.

b) $10 = -5t^2 + 22t + 15$

$5t^2 - 22t - 5 = 0$

$t = \frac{22 \pm \sqrt{(-22)^2 - 4(5)(-5)}}{2(5)}$

$= \frac{22 \pm \sqrt{584}}{10}$

$t = 4.62 \quad t = -0.22$

∴ The ball is above 10m for 4.62 s.

c) $5t^2 - 22t - 15 = 0$

$t = \frac{22 \pm \sqrt{(-22)^2 - 4(5)(-15)}}{2(5)}$

$= \frac{22 \pm \sqrt{784}}{10}$

$t = 5 \quad t = -0.6$

∴ It takes the ball 5s to hit the ground.

d) $t = \frac{-22}{2(-5)} = 2.2$ s

$h = -5(2.2)^2 + 22(2.2) + 15$

$= 39.2$ m

∴ The max ht. is 39.2 m.

3. $H = -0.011x^2 + 0.99x + 1.6$

a) $x = \frac{-0.99}{2(-0.011)} = 45$

$H = -0.011(45)^2 + .99(45) + 16$

$= 23.875$

∴ The max ht. is 23.875 m

b) $15 = -0.011x^2 + 0.99x + 1.6$

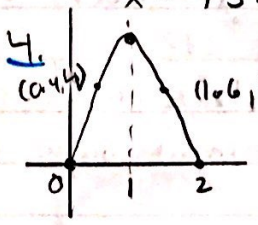
$0.011x^2 - 0.99x + 13.4 = 0$

$x = \frac{0.99 \pm \sqrt{(0.99)^2 - 4(0.011)(13.4)}}{2(0.011)}$

$= \frac{0.99 \pm \sqrt{0.3905}}{0.022}$

$x = 73.4$ m or $x = 16.6$ m

4.



$y = a(x-r)(x-s)$

$4 = a(0.4-0)(0.4-2)$

$4 = -0.64a$

$-6.25 = a$

$y = -6.25x(x-2)$

$y = -6.25(1)(1-2)$

$y = 6.25$

∴ Max ht. is 6.25 m.

5. Vertex: (28, 1024)
Point: (10, -4160)

x ⇒ price
y ⇒ profit

a) $y = a(x-h)^2 + k$
$-4160 = a(10-28)^2 + 1024$
$-5184 = 324a$
$-16 = a$
$y = -16(x-28)^2 + 1024$

b) Let $y = 0$.
$-16(x-28)^2 + 1024 = 0$
$-16(x-28)^2 = -1024$
$(x-28)^2 = 64$
$x - 28 = \pm 8$
$x = 8 + 28$ $\quad$ $x = -8 + 28$
$= 36$ $\quad$ $= 20$
∴ $20 and $36 will ensure she breaks even.

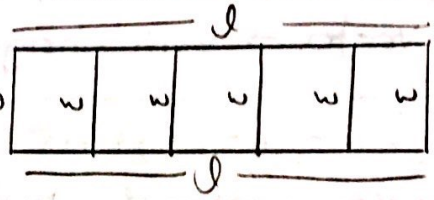
6. $h = -4.9t^2 + 21$

a) When does $h = 10.5$?
$-4.9t^2 + 21 = 10.5$
$-4.9t^2 = -10.5$
$\sqrt{t^2} = \sqrt{2.143}$
$t = 1.46$ seconds

b) When does $h = 0$?
$-4.9t^2 + 21 = 0$
$-4.9t^2 = -21$
$t^2 = 4.286$
$t = 2.07$ seconds

c) Gravity!

7.



$6w + 2l = 30$
$l = -3w + 15$
$A = lw$
$A = (-3w + 15)(w)$
$A = -3w^2 + 15w$
$= -3(w^2 - 5w + \frac{25}{4} - \frac{25}{4})$
$= -3(w^2 - 5w + \frac{25}{4}) + \frac{75}{4}$
$= -3(w - \frac{5}{2})^2 + 75/4$
∴ The width is 2.5m and the length is 7.5m, and the area is $18.75 m^2$

$c = (\frac{-5}{2})^2$

8. $P = 8x^2 - 112x + 570$

a) 570 (Let $x = 0$)

b) $x = \frac{112}{2(8)}$ $\quad$ $P = 8(7)^2 - 112(7) + 570$
$= 7$ $\quad$ $= 178$
∴ In 2006 there were 178 deer.

c) No, the minimum value is 178.

d) Let $x = 21$
$P = 8(21)^2 - 112(21) + 570$
$= 1746$ deer
Probably not because the graph is increasing rapidly by then.

9. $d = 0.1t^2 - 3.5t + 6$

a) $0.1t^2 - 3.5t + 6 = 0$
$t = \frac{3.5 \pm \sqrt{3.5^2 - 4(0.1)(6)}}{0.2}$
$= \frac{35 \pm \sqrt{11.85}}{0.2}$
$t = 33.2$ $\quad$ $t = 1.8$
∴ She is underwater for 31.4 seconds.

b) $t = \frac{33.2 + 1.8}{2}$
$= 17.5$ s
$d = 0.1(17.5)^2 - 3.5(17.5) + 6$
$= -24.63$ m
∴ She reaches a depth of 24.6m.

10. $C = 0.01x^2 - 1.5x + 93.25$

a) $x = \frac{1.5}{0.02}$

$= 75$ items

b) $0.01x^2 - 1.5x + 93.25 = 53$

$0.01x^2 - 1.5x + 40.25 = 0$

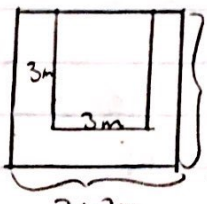
$x = \frac{1.5 \pm \sqrt{(1.5)^2 - 4(0.01)(40.25)}}{2(0.01)}$

$= \frac{1.5 \pm \sqrt{0.64}}{0.02}$

$x = 115 \quad x = 35$

A production rate between 35 and 115 items will keep cost below $53.

11. Let x be the width of the border.



$A_g = 9m^2$

$A_b = 9m^2$

$A_{bigsquare} - A_g = A_{border}$

$(3+x)(3+2x) - 9 = 9$

$9 + 9x + 2x^2 - 9 = 9$

$2x^2 + 9x - 9 = 0$

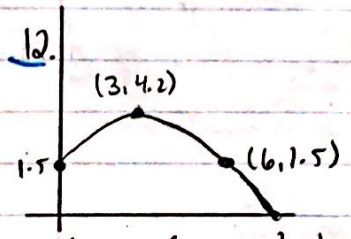
$x = \frac{-9 \pm \sqrt{(9)^2 - 4(2)(-9)}}{2(2)}$

$= \frac{-9 \pm \sqrt{153}}{4}$

$x = 0.84m \quad$ ~~$x = -5.34$~~

∴ The border is 0.84m wide.

12.



$h = a(x-h)^2 + k$

$1.5 = a(0-3)^2 + 4.2$

$-2.7 = 9a$

$-0.3 = a$

$h = -0.3(x-3)^2 + 4.2$

Let $h = 0$

$-0.3(x-3)^2 + 4.2 = 0$

$-0.3(x-3)^2 = -4.2$

$(x-3)^2 = 14$

$x = \sqrt{14} + 3 \quad$ or $\quad x = -\sqrt{14} + 3$

$= 6.74 \quad = -0.74$

∴ It hits the ground after 6.74m.

13. Let n be the first integer.

$(n)^2 + (n+2)^2 + (n+4)^2 = 980$

$n^2 + n^2 + 4n + 4 + n^2 + 8n + 16 = 980$

$3n^2 + 12n + 20 = 980$

$\frac{3n^2}{3} + \frac{12n}{3} - \frac{960}{3} = 0$

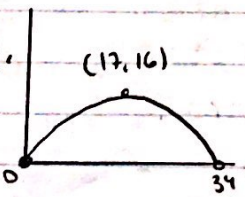
$n^2 + 4n - 320 = 0$

$(n+20)(n-16) = 0$

∴ $n = -20$ or $n = 16$

∴ The integers are $-20, -18, -16$ or $16, 18, 20$.

14.



a) $16 = a(17-0)(17-34)$

$\frac{-16}{289} = a$

$y = -\frac{16}{289}(x)(x-34)$

b) $y = -\frac{16}{289}x^2 + \frac{32}{17}x$

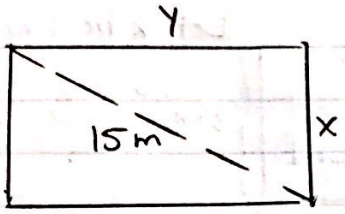
$\frac{16}{289}x^2 - \frac{32}{17}x + 3.3 = 0$

$x = \frac{\frac{32}{17} \pm \sqrt{(\frac{32}{17})^2 - 4(\frac{16}{289})(3.3)}}{2(\frac{16}{289})}$

$x = 32.1m$ or $x = 1.85m$

∴ She needs to be between 1.85m + 32.1m from it.

17.



① $x^2+y^2=15^2$ ② $x+y+15=36$

$x^2+y^2=225$ $x+y=21$

From ②: $x=21-y$

Sub in to ①: $(21-y)^2+y^2=225$

$441-42y+2y^2=225$

$2y^2-42y+216=0$

$2(y^2-21y+108)=0$

$y=\frac{21\pm\sqrt{(-21)^2-4(1)(108)}}{2(1)}$

$=\frac{21\pm\sqrt{9}}{2}$

$y=\frac{24}{2}$ or $y=\frac{18}{2}$

$\therefore y=12$ or 9

$\therefore$ The dimesions are 12m × 9m.

Chapter 6 Review

1a) $(2x-5)(3x+8)=0$

$x=5/2 \quad x=-8/3$

b) $x^2+12x+32=0$

$(x+8)(x+4)=0$

$x=-8, x=-4$

c) $3x^2-10x-8=0$

$3x^2-12x+2x-8=0$

$3x(x-4)+2(x-4)=0$

$(x-4)(3x+2)=0$

$x=4, x=-2/3$

d) $3x^2-5x+5=2x^2+4x-3$

$x^2-9x+8=0$

$(x-8)(x-1)=0$

$x=8, x=1$

e) $2x^2+5x-1=0$

$$x=\frac{-5\pm\sqrt{5^2-4(2)(-1)}}{2(2)}$$

$$=\frac{-5\pm\sqrt{33}}{4}$$

$x=0.19 \quad x=-2.69$

f) $5x(x-1)+5=7+x(1-2x)$

$5x^2-5x+5=7+x-2x^2$

$7x^2-6x-2=0$

$$x=\frac{6\pm\sqrt{6^2-4(7)(-2)}}{2(7)}$$

$$=\frac{6\pm\sqrt{92}}{14}$$

$x=1.11 \quad x=-0.26$

4e) $y=2(x^2+5x+\frac{25}{4}-\frac{25}{4})-12$

$y=2(x+\frac{5}{2})^2-\frac{25}{2}-12$

$y=2(x+\frac{5}{2})^2-49/2$

2. $d=0.002(2v^2+10v+3000)$

a) $d=0.002(2(12)^2+10(12)+3000)$

$=6.816\ m$

b) $15=0.002(2v^2+10v+3000)$

$7500=2v^2+10v+3000$

$2v^2+10v-4500=0$

$v^2+5v-2250=0$

$(v+50)(v-45)=0$

$\boxed{v=45}$

3a) $c=\left(\frac{8}{2}\right)^2=16$ b) $c=\left(\frac{-16}{2}\right)^2=64$ c) $c=\left(\frac{19}{2}\right)^2=\frac{361}{4}$

d) $c=\left(\frac{12}{2}\right)^2=36$ e) $c=\left(\frac{-5}{2}\right)^2=\frac{25}{4}$ f) $c=\left(\frac{-70}{2}\right)^2=1225$

4a) $y=x^2+8x-2$

$y=(x^2+8x+16-16)-2$

$y=(x^2+8x+16)-16-2$

$y=(x+4)^2-18$

b) $y=x^2-20x+95$

$=(x^2-20x+100-100)+95$

$=(x^2-20x+100)-100+95$

$=(x-10)^2-5$

c) $y=-3(x^2-4x)-2$

$=-3(x^2-4x+4-4)-2$

$=-3(x^2-4x+4)+12-2$

$=-3(x-2)^2+10$

d) $y=0.2(x^2-2x)+1$

$=0.2(x^2-2x+1-1)+1$

$=0.2(x^2-2x+1)-0.2+1$

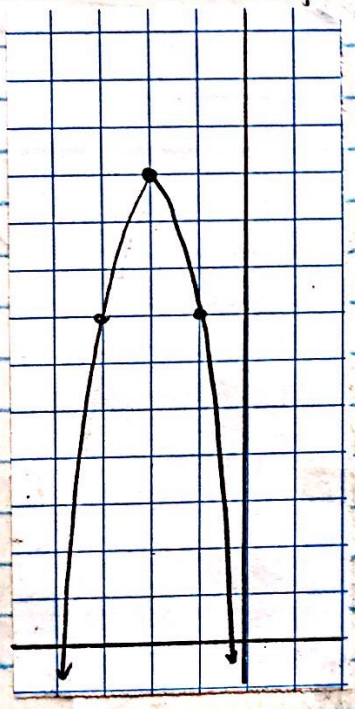
$=0.2(x-1)^2+0.8$

4f) $y = -4.9(x^2 + 4x) + 12$
$y = -4.9(x^2 + 4x + 4 - 4) + 12$
$= -4.9(x+2)^2 + 19.6 + 12$
$= -4.9(x+2)^2 + 31.6$

5a) $y = -3(x^2 + 4x) - 2$
$y = -3(x^2 + 4x + 4 - 4) - 2$
$= -3(x+2)^2 + 12 - 2$
$= -3(x+2)^2 + 10$

b) • reflect over x-axis
• v.s. by a factor of 3
• left 2 units + up 10 units.

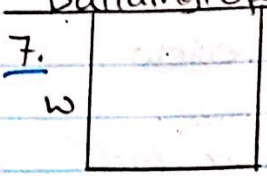
c)



6. $y = -0.2x^2 + 2.4x + 2$
$y = -0.2(x^2 - 12x) + 2$
$= -0.2(x^2 - 12x + 36 - 36) + 2$
$= -0.2(x-6)^2 + 9.2$
∴ The max height is 9.2m.

7.

Building/open side



$2w + l = 46 \rightarrow l = 46 - 2w$
$A = lw$
$= -2w^2 + 46w$
$= -2(w^2 - 23w + 132.25 - 132.25)$
$= -2(w - 11.5)^2 + 264.5$
∴ The max area is 264.5m

8a) $3x^2 - 4x - 10 = 0$
$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-10)}}{2(3)}$
$= \frac{4 \pm \sqrt{136}}{6}$
$x = 2.61 \quad x = -1.28$

b) $-4x^2 + 1 = -15$
$-4x^2 = -16$
$x^2 = 4$
$x = \pm 2$

c) $x^2 = 6x + 10$
$x^2 - 6x - 10 = 0$
$x = \frac{6 \pm \sqrt{(6)^2 - 4(1)(-10)}}{2(1)}$
$= \frac{6 \pm \sqrt{76}}{2}$
$x = 7.36 \quad x = -1.36.$

d) $(x-3)^2 - 4 = 0$
$(x-3)^2 = 4$
$x - 3 = \pm 2$
$x = 5, x = 1$

8e) $(2x+5)(3x-2) = (x+1)$

$6x^2 - 4x + 15x - 10 = x + 1$

$6x^2 + 11x - 10 = x + 1$

$6x^2 + 10x - 11 = 0$

$x = \frac{-10 \pm \sqrt{(10)^2 - 4(6)(-11)}}{2(6)}$

$= \frac{-10 \pm \sqrt{124}}{12}$

$x = 0.09 \quad x = -1.76$

f) $1.5x^2 - 6.1x + 1.1 = 0$

$x = \frac{6.1 \pm \sqrt{(-6.1)^2 - 4(1.5)(1.1)}}{2(1.5)}$

$= \frac{6.1 \pm \sqrt{30.61}}{3}$

$x = 3.88 \quad x = 0.19$

9. $h = -0.1x^2 + 2.4x + 8.1$

$0.1x^2 - 2.4x - 8.1 = -10$

$0.1x^2 - 2.4x + 1.9 = 0$

$x = \frac{2.4 \pm \sqrt{(-2.4)^2 - 4(0.1)(1.9)}}{2(0.1)}$

$= \frac{2.4 \pm \sqrt{5}}{0.2}$

$x = 23.18\text{m} \quad x = 0.82\text{m}$

∴ The balloon has travelled 0.82m or 23.18m.

10. $h = 0.0025x^2 - 0.9x + 120$

$0.0025x^2 - 0.9x + 120 = 50$

$0.0025x^2 - 0.9x + 70 = 0$

$x = \frac{0.9 \pm \sqrt{0.11}}{0.005}$

$x = 246.3 \quad x = 113.67$

∴ It is 246.3cm or 113.67cm.

11a) $D = (-5)^2 - 4(2)(1)$

$= 25 - 8$

$= 17$ ∴ 2 real roots

b) $D = (-2.1)^2 - 4(-3.5)(-1)$

$= -9.59$ ∴ no real roots

c) $D = (5)^2 - 4(1)(8)$

$= 25 - 32$

$= -7$ ∴ no real roots

d) $D = (0)^2 - 4(4)(-15)$

$= 240$ ∴ Two real roots

e) $5x^2 + 10x + 25 = -4x + 50$

$5x^2 + 14x - 25 = 0$

$D = (14)^2 - 4(5)(-25)$

$= 696$ ∴ Two real roots.

12a) 2 b) 2 c) $D = (8)^2 - 4(2)(14)$

d) $2x^2 - 10x + 7$ $= -48$ ∴ no roots

$D = (-10)^2 - 4(2)(7)$ e) $D = (-4)^2 - 4(-1.4)(-5.4)$

$= 44$ ∴ 2 roots $= -14.24$ ∴ no roots

13. $H = 3500 - 5t^2$

a) $H = 3500 - 5(10)^2$

$= 3000$ ∴ They have fallen 500m.

b) $1000 = 3500 - 5t^2$

$-2500 = -5t^2$

$500 = t^2$ ∴ They fell for

$22.36 = t$ 22.4 seconds.

14. $h = -0.008x^2 - 1.296x + 1075$

$x = \frac{1.296}{2(-0.008)}$ $h = -0.008(81)^2 - 1.296(81) + 1075$

$= -81$ $= 160\text{m}$

∴ The height is 160m.

Mitrey

15. Let x be the # of $0.50 increases.

$R = (5 + 0.5x)(300 - 20x)$

$R = -10x^2 + 50x + 1500$

$1562.50 = -10x^2 + 50x + 1500$

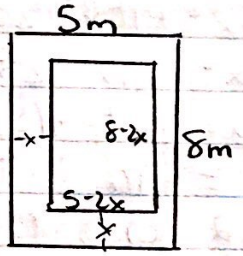
$10x^2 - 50x + 62.50 = 0$

$x = \frac{50 \pm \sqrt{0}}{20}$

$= 2.5$

∴ They should charge $7.50 to earn $1562.50.

16.



$A_{rug} = \frac{3}{4}(40)$

$= 30\ m^2$

$A_{room} = 40\ m^2$

Let x be the width of the strip.

$A_{rug} = 30$

$(5 - 2x)(8 - 2x) = 30$

$40 - 10x - 16x + 4x^2 = 30$

$4x^2 - 26x + 10 = 0$

$2x^2 - 13x + 5 = 0$

$x = \frac{13 \pm \sqrt{105}}{4}$

$x = 5.8$ | $x = 0.7$

∴ The strip is about 0.7m wide.

17. Let x be the larger #.

① $x - y = 12 \rightarrow x = 12 + y$

② $x^2 + y^2 = 1040$

$(12 + y)^2 + y^2 = 1040$

$y^2 + 24y + 144 + y^2 = 1040$

$2y^2 + 24y - 896 = 0$

$y^2 + 12y - 448 = 0$

$(y + 28)(y - 16) = 0$

∴ The integers are ±16 and ±28.