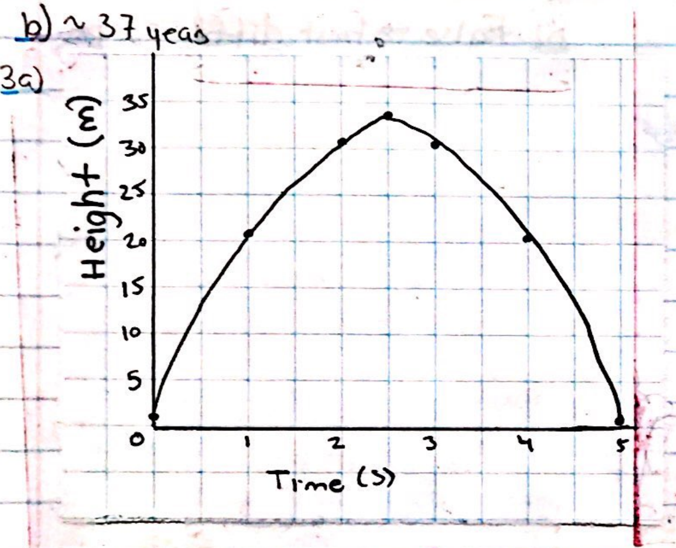
Chapter 3 - Graphs of Quadratic Relations

Getting Started

1. a → v
b → ii
c → i
d → iv
e → vi
f → iii

2a) ~58bpm
b) ~37 yeas

3a)



b) ~34.5m c) ~1.4s and ~3.6s

4a) $4(x+3)$
$= 4x+12$

b) $2x(x-5)$
$= 2x^2-10x$

c) $-3x^2(x-2)$
$= -3x^3+6x^2$

d) $4x(2x-3)+3x(7-5x)$
$= 8x^2-12x+21x-15x^2$
$= -7x^2+9x$

e) $7x^2(4x-7+2x^2)-x(3x^2-5x-2)$
$= 28x^3-49x^2+14x^4-3x^3+5x^2+2x$
$= 14x^4+25x^3-44x^2+2x$

f) $-4x(x^3-3x^2)+2x(5x^2-3x)-6x^3(x-3)$
$= -4x^4+12x^3+10x^3-6x^2-6x^4+18x^3$
$= -10x^4+40x^3-6x^2$

5a) $y=2x-3$
$y=2(1.5)-3$
$y=3-3$
$y=0$ $(1.5,0)$

b) $y=x^2$
$y=(-3)^2$
$y=9$
$(-3,9)$

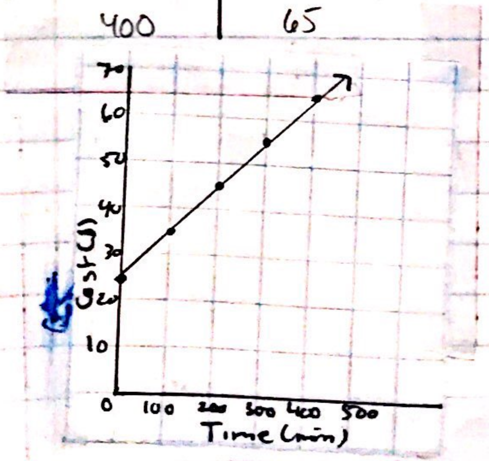
c) $y=x^2+2x-1$
$y=(4)^2+2(4)-1$
$y=16+8-1$
$y=23$
$(4,23)$

d) $y=(2x+1)(x-3)$
$=(2(2)+1)(2-3)$
$=(5)(-1)$
$=-5$
$(2,-5)$

6.

Time(min)	Cost($)
0	25
100	35
200	45
300	55
400	65

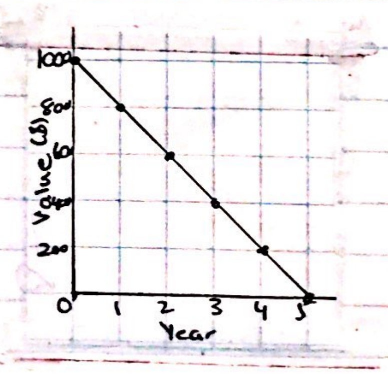
$y=0.10x+25$



7.

Year	Value
0	1000
1	800
2	600
3	400
4	200

$y=1000-200x$



8. $(-4,0)$ and $(0,-8)$

9a) $y=2x-3$
y-int: $(0,-3)$
x-int (let $y=0$)
$2x-3=0$
$2x=3$
$x=\frac{3}{2}$
$(\frac{3}{2},0)$

b) $x+y=5$
y-int: $(0,5)$
x-int: $(5,0)$

9c) $y = 2x + 5$

y-int: (0,5)

x-int (let y=0)

$2x + 5 = 0$

$2x = -5$

$x = -5/2$

$(-5/2, 0)$

d) $3x + 2y = 12$

y-int (let x=0)

$2y = 12$

$y = 6$ (0,6)

x-int (let y=0)

$3x = 12$

$x = 4$ (4,0)

e) $x = 2$

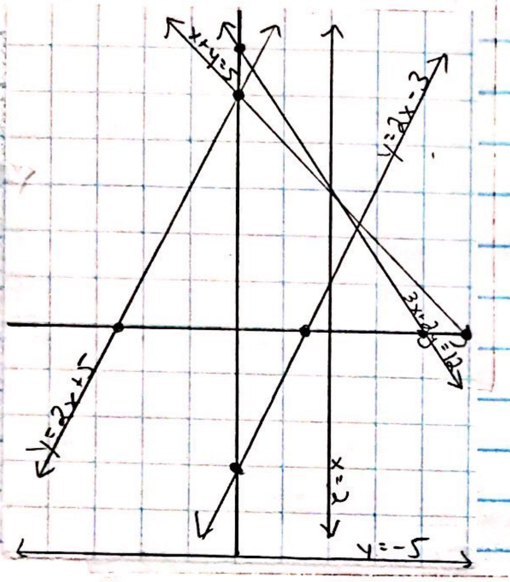
no y-int

x-int (2,0)

f) $y = -5$

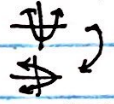
no x-int

y-int (0,-5)



10a) True

b) False → first diff/Δx = slope

c) False → 

d) False →

e) False → 

3.1 Exploring Quadratic Relations.

1. c, d, f are probably quadratic relations → smooth curve, max/min.

2a i) $y = 5x - 2 \rightarrow 1^{st}$ degree

ii) $y = x^2 - 6x + 4 \rightarrow 2^{nd}$ degree

iii) $y = x(x+4)$

$y = x^2 + 4x \rightarrow 2^{nd}$ degree

iv) $y = 2x^3 - 4x^2 + 5x - 1 \rightarrow 3^{rd}$ degree

b) ii and iii are parabolas

3. Let $x = 0$:

i) $y = -2$ $(0,-2)$

ii) $y = 4$ $(0,4)$

iii) $y = 0$ $(0,0)$

iv) $y = -1$ $(0,-1)$

4a)

x	y	
10	21	
		20
20	41	
		20
30	61	
		20
40	81	

∴ linear

b)

x	y		
1	4		
		3	
2	7		2
		5	
3	12		0
		5	
4	17		

∴ neither

c)

x	y		
5	-2		
		-1	
6	-3		-1
		-2	
7	-5		-1
		-3	
8	-8		

∴ quadratic

d)

x	y		
0	1		
		-2	
1	-1		10
		8	
2	7		-26
		-18	
3	-11		

∴ neither

e)

x	y		
0	-2		
		1	
1	-1		6
		7	
2	6		12
		19	
3	25		

∴ neither

f)

x	y		
0	1		
		1	
1	2		1
		2	
2	4		2
		4	
3	8		4
		8	
4	16		

∴ neither

5a)

x	y		
-3	2.5		
		2.5	
-2	5.0		-1
		1.5	
-1	6.5		-1
		0.5	
0	7.0		

∴ opens down

5b)

x	y		
-2	0		
		-5	
-1	-5		10
		5	
0	0		10
		15	
1	15		10
		25	
2	40		

∴ opens up

c)

x	y		
-2	-3		
		6	
-1	3		-4
		2	
0	5		-4
		-2	
1	3		-4
		-6	
2	-3		

∴ opens down

d)

x	y		
0	-1		
		5	
1	4		6
		11	
2	15		6
		17	
3	32		6
		23	
4	55		

∴ opens up

Hilary

6a) $y = x^2 - 1$ b) $y = \underset{\uparrow}{-}x^2 + 5x$ c) $y = \underset{\uparrow}{-\frac{1}{2}}x^2 + 6x - 4$

∘ opens up — Opens down — opens down

d) $y = \underset{\uparrow}{\frac{1}{2}}x^2 - 9x + 6$

Opens up

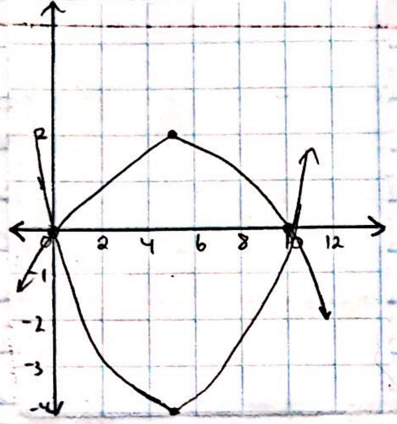
7. If $a = 0$, $y = (0)x^2 + bx + c$

$y = bx + c$ ← linear relation!

3.2 Properties of Graphs of Quadratic Relations

1. Part	y-intercept	zeros	vertex	eqn of axis	Min/max	2.
a)	(0, 0)	(0,0) + (-4,0)	(-2, 4)	$x = -2$	max, $y = 4$	a
b)	(0, -3)	(-2,0) + (2,0)	(0, -3)	$x = 0$	min, $y = -3$	b
c)	(0, 0)	(0,0) + (4,0)	(2, 4)	$x = 2$	max, $y = 4$	c

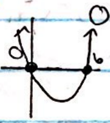
3.



4a i) (3, 2)
ii) (0,0) + (6,0)
iii) $x = 3$
iv) negative, opens down

b i) (2, -3)
ii) (0, 0) + (4, 0)
iii) $x = 2$
iv) positive, opens up

5.



a) minimum (opens up)
b) negative (below x-axis)
c) $x = 3$

6a i) $x = -4$
ii) (-4, -8)
iii) (0, 0)
iv) (0, 0) + (-8, 0)
v) $y = -8$ (min)

b i) $x = 2$
ii) (2, 2)
iii) (0, 0)
iv) (0, 0) + (4, 0)
v) $y = 2$ (max)

c i) $x = 0$
ii) (0, 8)
iii) (0, 8)
iv) (-6, 0) + (6, 0)
v) $y = 8$ (max)

7a) $y = x^2 + 2$*

x	$y = x^2 + 2$	(x, y)
-2	$(-2)^2 + 2 = 6$	(-2, 6)
-1	$(-1)^2 + 2 = 3$	(-1, 3)
0	$(0)^2 + 2 = 2$	(0, 2)
1	$(1)^2 + 2 = 3$	(1, 3)
2	$(2)^2 + 2 = 6$	(2, 6)

b) $y = -x^2 - 1$*

x	$y = -x^2 - 1$	(x, y)
-2	$-(-2)^2 - 1 = -5$	(-2, -5)
-1	$-(-1)^2 - 1 = -2$	(-1, -2)
0	$-(0)^2 - 1 = -1$	(0, -1)
1	$-(1)^2 - 1 = -2$	(1, -2)
2	$-(2)^2 - 1 = -5$	(2, -5)

c) $y = x^2 - 2x$*

x	$y = x^2 - 2x$	(x, y)
-2	$(-2)^2 - 2(-2) = 8$	(-2, 8)
-1	$(-1)^2 - 2(-1) = 3$	(-1, 3)
0	$(0)^2 - 2(0) = 0$	(0, 0)
1	$(1)^2 - 2(1) = -1$	(1, -1)
2	$(2)^2 - 2(2) = 0$	(2, 0)

i) axis of symmetry:	$x = 0$	$x = 0$	$x = 1$
ii) vertex:	(0, 2)	(0, -1)	(1, -1)
iii) y-intercept:	(0, 2)	(0, -1)	(0, 0)
iv) zeros:	no zeros	no zeros	(0, 0) and (2, 0)
v) max/min value:	$y = 2$ (min)	$y = -1$ (max)	$y = -1$ (min)

7d) $y = -x^2 + 4x$ ★

x	$y = -x^2+4x$	(x, y)
-2	$-(-2)^2+4(-2) = -12$	(-2, -12)
-1	$-(-1)^2+4(-1) = -5$	(-1, -5)
0	$-(0)^2+4(0) = 0$	(0, 0)
1	$-(1)^2+4(1) = 3$	(1, 3)
2	$-(2)^2+4(2) = 4$	(2, 4)
3	$-(3)^2+4(3) = 3$	(3, 3)

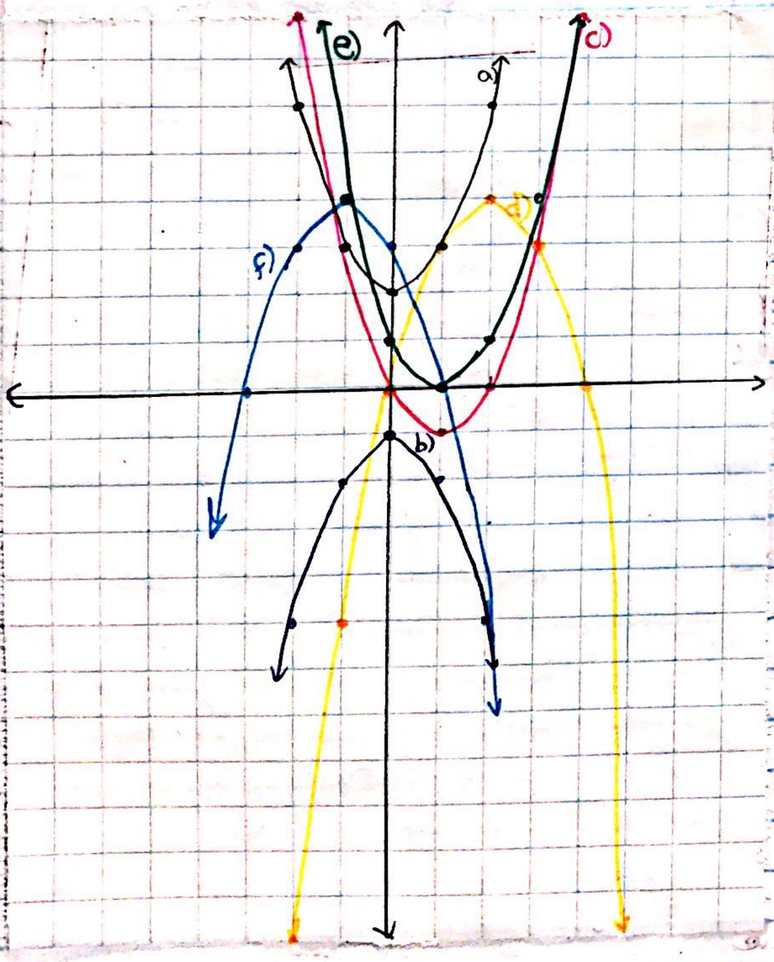
e) $y = x^2 - 2x + 1$ ★

x	$y = x^2-2x+1$	(x, y)
-2	$(-2)^2-2(-2)+1 = 9$	(-2, 9)
-1	$(-1)^2-2(-1)+1 = 4$	(-1, 4)
0	$(0)^2-2(0)+1 = 1$	(0, 1)
1	$(1)^2-2(1)+1 = 0$	(1, 0)
2	$(2)^2-2(2)+1 = 1$	(2, 1)

f) $y = -x^2 - 2x + 3$ ★

x	$y = -x^2-2x+3$	(x, y)
-2	$-(-2)^2-2(-2)+3 = 3$	(-2, 3)
-1	$-(-1)^2-2(-1)+3 = 4$	(-1, 4)
0	$-(0)^2-2(0)+3 = 3$	(0, 3)
1	$-(1)^2-2(1)+3 = 0$	(1, 0)
2	$-(2)^2-2(2)+3 = -5$	(2, -5)

i) axis of symmetry:	x = 2	x = 0	x = -1
ii) vertex:	(2, 4)	(1, 0)	(-1, 4)
iii) y-intercept:	(0, 0)	(0, 1)	(0, 3)
iv) zeros:	(0, 0) + (4, 0)	(1, 0)	(1, 0) + (-3, 0)
v) max or min value:	y = 4 (max)	y = 0 (min)	y = 4 (max)



9a) $x = \frac{3+9}{2}$

$x = 6$

b) $x = \frac{-18+7}{2}$

$x = -\frac{11}{2}$

c) $x = \frac{-5.25+3.75}{2}$

$x = \frac{-1.5}{2}$

$x = -0.75$

d) $x = \frac{-4\frac{1}{2}+(-1\frac{1}{2})}{2}$

$x = \frac{-6}{2}$

$x = -3$

10. $x = \frac{-1+5}{2}$

Sub $\boxed{x = 2}$ into $y = 4x^2 - 16x + 21$

$y = 4(2)^2 - 16(2) + 21$

$= 4(4) - 32 + 21$

$= 16 - 32 + 21$

$\boxed{y = 5}$

∴ Vertex is (2, 5)

11a) disagree – see ← graph

b) agree

c) disagree!

12. $h = 20t - 5t^2$

a) $20t - 5t^2 = 0$

$-5t(t-4) = 0$

∴ The ball hits the ground after 4 seconds.

b) $h = 20(2) - 5(2)^2$

$= 40 - 20$

$= 20$

$(2, 20)$

d) The ball reaches a max height of 20m after 2seconds.

13. $P = -60x^2 + 120x$

$P = -60x(x-2)$

c) Breaks even at 0 and 2000 sales

b) Sell 1000 to max. profit

a) $P = -60(1)^2 + 120(1)$

$= -60 + 120$

$= 60$

Max profit is \$60 000

14. $y = -5x^2 + 500$

a) Let $x = 0$

$y = 500$ ← 500m above

b) Let $y = 0$

$-5x^2 + 500 = 0$

$-5x^2 = -500$

$x^2 = 100$

$x = 10$ seconds

c) Let $x = 6$

$y = -5(6)^2 + 500$

$= -180 + 500$

$= 320$ m

d) $100 = -5x^2 + 500$

$-400 = -5x^2$

$80 = x^2$

$\sim 8.9s = x$

15. $P = -5x^2 + 60x - 135$

$P = -7x^2 + 70x - 63$

Find where they are equal.

$-5x^2 + 60x - 135 = -7x^2 + 70x - 63$

$2x^2 - 10x - 72 = 0$

$x^2 - 5x - 36 = 0$

$(x+4)(x-9) = 0$

∴ They cross when 900 000 players are sold.

When $x = 10$, last year:

$P = -5(10)^2 + 60(10) - 135$

$= -\$35000$ (loss)

This year:

$P = -7(10)^2 + 70(10) - 63$

$= -\$63 000$

∴ If they sell < 900 000 players each year, profit has increased this year. If they sell >900 000 players, profit is less this year.

16a) If 'a' is ⊕, minimum

If 'a' is ⊖, maximum

b) y-coord. of vertex is the max/min value

17a) i)

		x	y
		8	-2
	-6	2	-1
4	-2	0	0
4	2	2	1
4	6	8	2

ii)

x	y		
-2	1/4	1/4	
-1	1/2	1/2	$\frac{1}{4}$
0	1	1	$\frac{1}{2}$
1	2	2	1
2	4		

17a) iii)

x	y		
-2	-8	7	
-1	-1	1	-6
0	0	1	0
1	1	7	6
2	8		

iv)

x	y		
-2	32	-30	
-1	2	-2	28
0	0	2	4
1	2	30	28
2	32		

b) No! They do not

c) have $y = x^2$ in them!

Hilary

3.3 Factored Form of a Quadratic Relation

1a) $y = -2x(x+3)$ *

i + ii) $-2x(x+3) = 0$

$-2x = 0 \quad x+3 = 0$

$x = 0 \quad x = -3$

iii) Let $x = 0$

$y = 0 \quad (0,0)$

iv) $x = \frac{0-3}{2}$

$x = -3/2$

v) $y = -2(-3/2)(-3/2 + 3)$

$= -3(3/2)$

$= -9/2$

$(-3/2, -9/2)$

vi) yes, it is second degree.

b) $y = (x-3)(x+1)$ *

i + ii) $(x-3)(x+1) = 0$

$x-3 = 0 \quad x+1 = 0$

$x = 3 \quad x = -1$

$(3,0) \quad (-1,0)$

iii) Let $x = 0$

$y = (0-3)(0+1)$

$= (-3)(1)$

$= -3 \quad (0,-3)$

iv) $x = \frac{3-1}{2}$

$x = 1$

v) $y = (1-3)(1+1)$

$y = -4 \quad (1,-4)$

vi) yes!

c) $y = 2(x-1)(x+2)$ *

i + ii) $2(x-1)(x+2) = 0$

$x-1 = 0 \quad x+2 = 0$

$x = 1 \quad x = -2$

$(1,0) \quad (-2,0)$

iii) Let $x = 0$

$y = 2(0-1)(0+2)$

$= -4$

iv) $x = \frac{1-2}{2}$

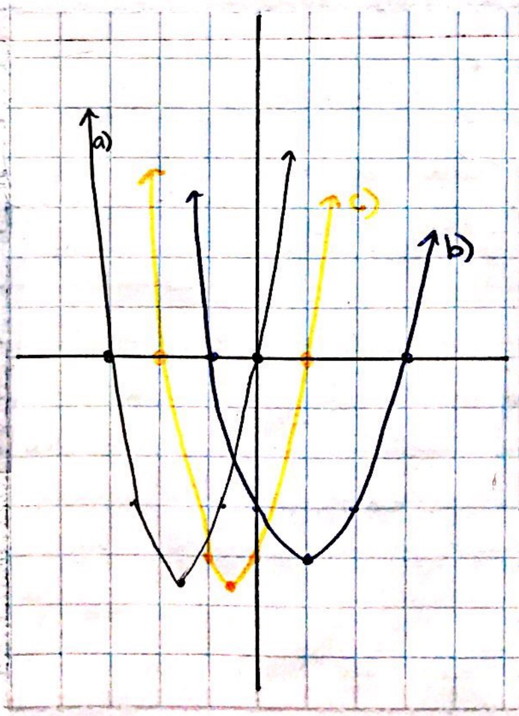
$x = -\frac{1}{2}$

v) $y = 2(-\frac{1}{2}-1)(-\frac{1}{2}+2)$

$= 2(-\frac{3}{2})(\frac{3}{2})$

$= -\frac{9}{2} \quad (-\frac{1}{2}, -\frac{9}{2})$

vi) yes!



2a) ii
b) iv
c) i
d) vi
e) v
f) iii

3. $y = a(x-r)(x-s)$

$5 = a(3-2)(3+6)$

$5 = a(1)(9)$

$5 = 9a$

$\frac{5}{9} = a$

$y = \frac{5}{9}(x-2)(x+6)$

4a) $y = (x-3)(x+3)$ *

y-int: Let $x=0$

$y = (-3)(3)$

$y = -9$

Zeros: $x = \pm 3$

a of s: $x = 0$

Vertex: $(0,-9)$

b) $y = (x+2)(x+2)$ *

y-int: Let $x = 0$

$y = (0+2)(0+2)$

$y = 4$

Zeros: $x = -2$

a of s: $x = -2$

Vertex: $(-2,0)$

4c) $y = (x-2)(x-2)$ *

y-int: $y = (-2)(-2)$

$= 4$

Zeros: $x = 2$

a of s: $x = 2$

Vertex: $(2,0)$

d) $y = -(x-2)(x+2)$ *

y-int: $y = -(-2)(2)$

$= 4$

Zeros: $x = \pm 2$

a of s: $x = 0$

Vertex: $(0,4)$

5.

4e) $y = 2(x+3)^2$ ★

y-int: $y = 2(3)^2$

$y = 18$

zeros: $x = -3$

a of s: $x = -3$

vertex: $(-3, 0)$

f) $y = -4(x-4)^2$ ★

y-int: $y = -4(-4)^2$

$y = -64$

zeros: $x = 4$

a of s: $x = 4$

vertex: $(4, 0)$

6a) $y = a(x-r)(x-s)$

$1 = a(0-4)(0-2)$

$1 = 8a$

$\frac{1}{8} = a$

$y = \frac{1}{8}(x-4)(x-2)$

b) $y = a(x-r)(x-s)$

$-1 = a(0-4)(0+2)$

$-1 = -8a$

$\frac{1}{8} = a$

$y = \frac{1}{8}(x-4)(x+2)$

c) Vertex: $(2.5, \mp 10)$

$-10 = a(2.5-5)(2.5-0)$

$-10 = a(-2.5)(2.5)$

$-10 = -6.25a$

$1.6 = a$

$y = \frac{8}{5}(x-5)(x-0)$

OR $y = \frac{8}{5}x(x-5)$

d) Vertex: $(1, 6)$

$6 = a(1-5)(1+3)$

$6 = a(-4)(4)$

$6 = -16a$

$-\frac{3}{8} = a$

$y = -\frac{3}{8}(x-5)(x+3)$

6e) one zero, so r = s

$-10 = a(0-5)(0-5)$

$-10 = 25a$

$-\frac{2}{5} = a$

$y = -\frac{2}{5}(x-5)(x-5)$

OR $y = -\frac{2}{5}(x-5)^2$

7a) zeros: ± 40

a of s: $x = 0$

vertex: $(\overset{x}{0}, \overset{y}{40})$

$40 = a(0-40)(0+40)$

$40 = -1600a$

$-\frac{1}{40} = a$

$y = -\frac{1}{40}(x-40)(x+40)$

b) zeros: $(10, 0), (30, 0)$

a of s: $x = 20$

vertex: $(20, -10)$

$-10 = a(20-10)(20-30)$

$-10 = a(10)(-10)$

$-10 = -100a$

$\frac{1}{10} = a$ $y = \frac{1}{10}(x-10)(x-30)$

7c) zeros: $(1, 0), (-4, 0)$

a of s: $x = -\frac{3}{2}$

vertex: $(-\frac{3}{2}, -2)$

$-2 = a(-\frac{3}{2}-1)(-\frac{3}{2}+4)$

$-2 = a(-\frac{5}{2})(\frac{5}{2})$

$-2 = -\frac{25}{4}a$

$\frac{8}{25} = a$ $y = \frac{8}{25}(x-1)(x+4)$

d) zeros: $(-1, 0), (-5, 0)$

a of s: $x = -3$

vertex: $(-3, \frac{7}{2})$

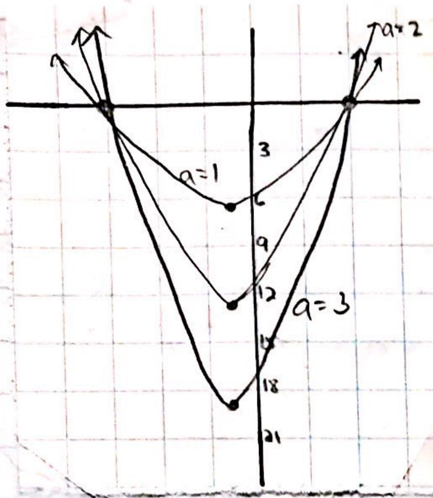
$\frac{7}{2} = a(-3+1)(-3+5)$

$\frac{7}{2} = a(-2)(2)$

$-\frac{7}{8} = a$

$y = -\frac{7}{8}(x+1)(x+5)$

8. $y = 3(x-2)(x+3)$



a) Vertex:

$y = 3(-\frac{1}{2}-2)(-\frac{1}{2}+3)$

$= -18.75$

$(-\frac{1}{2}, -18\frac{3}{4})$

b) If $a = 2$:

$y = 2(-2\frac{1}{2})(2\frac{1}{2})$

$= -12\frac{1}{2}$

$(-\frac{1}{2}, -12\frac{1}{2})$

If $a = 1$

$y = (-2\frac{1}{2})(2\frac{1}{2})$

$= -6\frac{1}{4}$

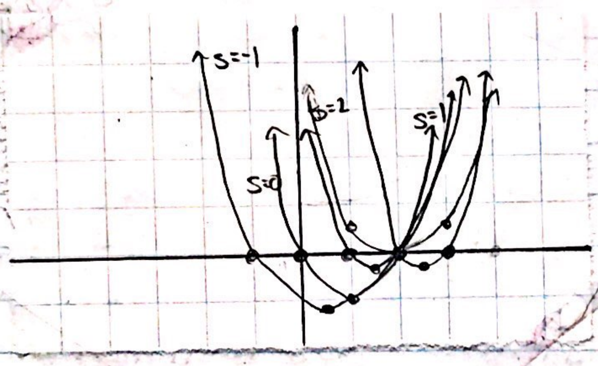
$(-\frac{1}{2}, -6\frac{1}{4})$

If $a = 0$:

$y = 0$. → It's the x-axis.

★ For all of the ⊖ values of a, the graph would open down, so the vertex would be: $(-\frac{1}{2}, 18\frac{3}{4})$, $(-\frac{1}{2}, 12\frac{1}{2})$, $(-\frac{1}{2}, 6\frac{1}{4})$

9.



a) a of s: $x = 2\frac{1}{2}$

vertex: $y = (2\frac{1}{2}-2)(2\frac{1}{2}-3)$

$= -\frac{1}{4}$

$(2\frac{1}{2}, -\frac{1}{4})$

b) If $s = 2$:

$y = (x-2)(x-2)$, so $r=s$ and the vertex is at $(2,0)$

If $s=0$:

$y = x(x-2)$

vertex: $(1,-1)$

If $s=-1$:

$y = (x-2)(x+1)$

Vertex: $(\frac{1}{2}, -1\frac{1}{4})$

If $s=1$:

$y = (x-2)(x-1)$

vertex: $(1\frac{1}{2}, -\frac{1}{4})$

As the zeros get further apart, the vertex gets lower!

10. $r=-3$, $s=5$, $x=0$, $y=-75$

a) $y = a(x-r)(x-s)$

$-75 = a(0+3)(0-5)$

$\frac{-75}{-15} = \frac{-15a}{-15}$

$5 = a$

$y = 5(x+3)(x-5)$

b) $x = \frac{-3+5}{2}$

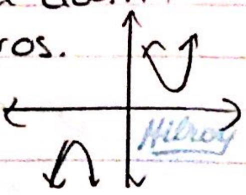
$x = 1$

$y = 5(1+3)(1-5)$

$= -80$

Vertex: $(1, -80)$

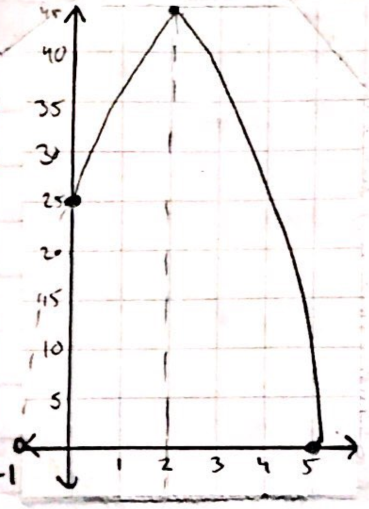
11. $y = a(x-r)(x-s)$ cannot be used if a parabola doesn't have zeros.



12.

a)

b)



c) $x = -1$

d) $(5,0) + (-1,0)$

e) $(2, 45)$

f) $45 = a(2-5)(2+1)$

$45 = a(-3)(3)$

$45 = -9a$

$-5 = a$

$y = -5(x-5)(x+1)$

g) (5,0) is when the ball hits the ground. (-1,0) doesn't really exist (time travel?)

13a) $5 = a(0+250)(0-50)$

$5 = -12500a$

$-\frac{1}{2500} = a$

$y = -\frac{1}{2500}(x+250)(x-50)$

b) The price should decrease by $100.

15. Let x be the number of price increases.

Revenue = price × quantity

a) $y = (5+0.1x)(700-10x)$

b) Zeros:

$0.1x + 5 = 0$ $-10x + 700 = 0$

$0.1x = -5$ $-10x = -700$

$x = -50$ $x = 70$

a of s:

$x = \frac{-50+70}{2}$

$= 10$

Vertex: $y = (5+0.1(10))(700-10(10))$

$= (6)(600)$

$= \$3600$

14. Let x be the # of price increases.

Revenue = price × quantity

$y = (10+x)(80-5x)$

Zeros:

$x + 10 = 0$ $-5x + 80 = 0$

$x = -10$ $-5x = -80$

$x = 16$

a of s: $x = \frac{-10+16}{2}$

$x = 3$

∴ He should increase his price 3 times + sell CDs for $13.

∴ his maximum revenue is $3600

c) He sells 600 packages of batteries to maximize revenue.

3.4 Expanding Quadratic Expressions

1a) $(x+5)(x+1)$
$= x^2+6x+5$

b) $(x-2)(x-2)$
$= x^2-4x+4$

2.

<table>
<tr><th>Expression</th><th>Area Diagram</th><th>Expanded + Simplified</th></tr>
<tr><td>a) $(x+1)(x+6)$</td><td><table><tr><td></td><td>x</td><td>1</td></tr><tr><td>x</td><td>x^2</td><td>$1x$</td></tr><tr><td>6</td><td>$6x$</td><td>6</td></tr></table></td><td>x^2+7x+6</td></tr>
<tr><td>b) $(x+1)(x-4)$</td><td><table><tr><td></td><td>x</td><td>1</td></tr><tr><td>x</td><td>x^2</td><td>$1x$</td></tr><tr><td>-4</td><td>$-4x$</td><td>-4</td></tr></table></td><td>x^2-3x-4</td></tr>
<tr><td>c) $(x-2)(x+2)$</td><td><table><tr><td></td><td>x</td><td>-2</td></tr><tr><td>x</td><td>x^2</td><td>$-2x$</td></tr><tr><td>2</td><td>$2x$</td><td>-4</td></tr></table></td><td>x^2-4</td></tr>
<tr><td>d) $(x-3)(x-4)$</td><td><table><tr><td></td><td>x</td><td>-3</td></tr><tr><td>x</td><td>x^2</td><td>$-3x$</td></tr><tr><td>-4</td><td>$-4x$</td><td>$+12$</td></tr></table></td><td>$x^2-7x+12$</td></tr>
<tr><td>e) $(x+2)(x+4)$</td><td><table><tr><td></td><td>x</td><td>2</td></tr><tr><td>x</td><td>x^2</td><td>$2x$</td></tr><tr><td>4</td><td>$4x$</td><td>8</td></tr></table></td><td>x^2+6x+8</td></tr>
<tr><td>f) $(x-2)(x-6)$</td><td><table><tr><td></td><td>x</td><td>-2</td></tr><tr><td>x</td><td>x^2</td><td>$-2x$</td></tr><tr><td>-6</td><td>$-6x$</td><td>$+12$</td></tr></table></td><td>$x^2-8x+12$</td></tr>
</table>

3a) $(m+3)(m+2)$
$= m^2 + 2m + 3m + 6$
b) $(k-2)(k+1)$
$= k^2 + k - 2k - 2$
c) $(r+4)(r-3)$
$= r^2 - 3r + 4r - 12$
d) $(x-5)(x-2)$
$= x^2 - 2x - 5x + 10$
e) $(2n+1)(3n-2)$
$= 6n^2 - 4n + 3n - 2$
f) $(5m-2)(m-3)$
$= 5m^2 - 15m - 2m + 6$

4a) $(x+2)(x+5)$
$= x(x+5) + 2(x+5)$
$= x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$
b) $(x+2)(x+1)$
$= x(x+1) + 2(x+1)$
$= x^2 + 1x + 2x + 2$
$= x^2 + 3x + 2$
c) $(x+2)(x-3)$
$= x(x-3) + 2(x-3)$
$= x^2 - 3x + 2x - 6$
$= x^2 - 1x - 6$

d) $(x+2)(x-1)$
$= x(x-1) + 2(x-1)$
$= x^2 - 1x + 2x - 2$
$= x^2 + 1x - 2$
e) $(x-4)(x-2)$
$= x(x-2) - 4(x-2)$
$= x^2 - 2x - 4x + 8$
$= x^2 - 6x + 8$
f) $(x-5)(x-3)$
$= x(x-3) - 5(x-3)$
$= x^2 - 3x - 5x + 15$
$= x^2 - 8x + 15$

5a) $(5x+2)(x+2)$
$= 5x(x+2) + 2(x+2)$
$= 5x^2 + 10x + 2x + 4$
$= 5x^2 + 12x + 4$
b) $(x+2)(4x+1)$
$= x(4x+1) + 2(4x+1)$
$= 4x^2 + 1x + 8x + 2$
$= 4x^2 + 9x + 2$
c) $(x-2)(7x+3)$
$= x(7x+3) - 2(7x+3)$
$= 7x^2 + 3x - 14x - 6$
$= 7x^2 - 11x - 6$
d) $(3x-2)(x+1)$
$= 3x(x+1) - 2(x+1)$
$= 3x^2 + 3x - 2x - 2$
$= 3x^2 + 1x - 2$
e) $(x-2)(4x+6)$
$= x(4x+6) - 2(4x+6)$
$= 4x^2 + 6x - 8x - 12$
$= 4x^2 - 2x - 12$
f) $(7x-5)(7x+5)$
$= 7x(7x+5) - 5(7x+5)$
$= 49x^2 + 35x - 35x + 25$
$= 49x^2 - 25$

6a) $(x+3)(x-3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$
b) $(x+6)(x-6)$
$= x^2 + 6x - 6x - 36$
$= x^2 - 36$
c) $(2x-1)(2x+1)$
$= 4x^2 + 2x - 2x - 1$
$= 4x^2 - 1$
d) $(3x-3)(3x+3)$
$= 9x^2 + 9x - 9x - 9$
$= 9x^2 - 9$
e) $(4x-6)(4x+6)$
$= 16x^2 + 24x - 24x - 36$
$= 16x^2 - 36$
f) $(7x-5)(7x+5)$
$= 49x^2 + 35x - 35x - 25$
$= 49x^2 - 25$

7f) $(-3d+5)^2$
$= (-3d+5)(-3d+5)$
$= 9d^2 - 15d - 15d + 25$
$= 9d^2 - 30d + 25$

7a) $(x+1)^2$
$= (x+1)(x+1)$
$= x^2 + 1x + 1x + 1$
$= x^2 + 2x + 1$
b) $(a+4)^2$
$= (a+4)(a+4)$
$= a^2 + 4a + 4a + 16$
$= a^2 + 8a + 16$
c) $(c-1)^2$
$= (c-1)(c-1)$
$= c^2 - c - c + 1$
$= c^2 - 2c + 1$
d) $(5y-2)^2$
$= (5y-2)(5y-2)$
$= 25y^2 - 10y - 10y + 4$
$= 25y^2 - 20y + 4$
e) $(6z-5)^2$
$= (6z-5)(6z-5)_{25}$
$= 36z^2 - 30z^{-30z}$
$= 36z^2 - 60z + 25$

8a) $A = l \times w$
$= (2m+3)(4m-4)$
$= 8m^2 - 8m + 12m - 12$
$= 8m^2 + 4m - 12$

b) $A = l \times w$
$= (3m+2)(3m+2)$
$= 9m^2 + 6m + 6m + 4$
$= 9m^2 + 12m + 4$

c) $A = b \times h$
$= (2x-4)(5x+3)$
$= 10x^2 + 6x - 20x - 12$
$= 10x^2 - 14x - 12$

d) $A = \frac{a+b}{2} h$
$= \left(\frac{x+1+3x+1}{2}\right)(2x+2)$
$= \left(\frac{4x+2}{2}\right)(2x+2)$
$= (2x+1)(2x+2)$
$= 4x^2 + 4x + 2x + 2$
$= 4x^2 + 6x + 2$

9a) $4(x-6)(x+7)$
$= 4(x^2 + 7x - 6x - 42)$
$= 4(x^2 + x - 42)$
$= 4x^2 + 4x - 168$

b) $-(x+3)(4x-1)$
$= -(4x^2 - x + 12x - 3)$
$= -(4x^2 + 11x - 3)$
$= -4x^2 - 11x + 3$

c) $6x(x+1)^2$
$= 6x(x+1)(x+1)$
$= 6x(x^2 + x + x + 1)$
$= 6x(x^2 + 2x + 1)$
$= 6x^3 + 12x^2 + 6x$

d) $(x+4)(x-2) + (x-1)(x+5)$
$= x^2 - 2x + 4x - 8 + x^2 + 5x - x + 5$
$= 2x^2 + 6x - 13$

e) $(4x-1)(4x+1) - (x+3)^2$
$= 16x^2 + 4x - 4x - 1 - (x+3)(x+3)$
$= 16x^2 - 1 - (x^2 + 3x + 3x + 9)$
$= 16x^2 - 1 - x^2 - 6x - 9$
$= 15x^2 - 6x - 10$

10f) $(9x-7y)^2$
$= (9x-7y)(9x-7y)$
$= 81x^2 - 63xy - 63xy + 49y^2$
$= 81x^2 - 126xy + 49y^2$

9f) $2(3x+4)^2 - 3(x-2)^2$
$= 2(3x+4)(3x+4) - 3(x-2)(x-2)$
$= 2(9x^2 + 12x + 12x + 16) - 3(x^2 - 2x - 2x + 4)$
$= 2(9x^2 + 24x + 16) - 3(x^2 - 4x + 4)$
$= 18x^2 + 48x + 32 - 3x^2 + 12x - 12$
$= 15x^2 + 60x + 20$

10a) $(x+y)(2x+3y)$
$= x(2x+3y) + y(2x+3y)$
$= 2x^2 + 3xy + 2xy + 3y^2$
$= 2x^2 + 5xy + 3y^2$

b) $(x+2y)(3x+y)$
$= 3x^2 + xy + 6xy + 2y^2$
$= 3x^2 + 7xy + 2y^2$

c) $(3x-2y)(5x+4y)$
$= 15x^2 + 12xy - 10xy - 8y^2$
$= 15x^2 + 2xy - 8y^2$

d) $(8x-y)(7x+2y)$
$= 56x^2 + 16xy - 7xy - 2y^2$
$= 56x^2 + 9xy - 2y^2$

e) $(6x-5y)(6x+5y)$
$= 36x^2 - 25y^2$

11a) $y = a(x+2)(x-4)$ Point: $(0,-8)$

$-8 = a(0+2)(0-4)$

$-8 = -8a$

$1 = a$

$y = (x+2)(x-4)$

$y = x^2 - 4x + 2x - 8$

$y = x^2 - 2x - 8$

b) $y = a(x+4)(x+2)$ Point: $(0,-8)$

$-8 = a(0+4)(0+2)$

$-8 = 8a$

$-1 = a$

$y = -(x+4)(x+2)$

$y = -(x^2 + 2x + 4x + 8)$

$y = -x^2 - 6x - 8$

c) $y = a(x-0)(x-4)$ Point: $(2,-8)$

$-8 = a(2-0)(2-4)$

$-8 = a(2)(-2)$

$-8 = -4a$

$2 = a$

$y = 2(x)(x-4)$

$y = 2x(x-4)$

$y = 2x^2 - 8x$

d) $y = a(x-2)(x-6)$ Point: $(4,2)$

$2 = a(4-2)(4-6)$

$2 = a(2)(-2)$

$2 = -4a$

$-\frac{1}{2} = a$

$y = -\frac{1}{2}(x-2)(x-6)$

$y = -\frac{1}{2}(x^2 - 6x - 2x + 12)$

$y = -\frac{1}{2}(x^2 - 8x + 12)$

$y = -\frac{1}{2}x^2 + 4x - 6$

12a) $5 = a(3+1)(3-7)$

$5 = a(4)(-4)$

$5 = -16a$ Opens down

$-\frac{5}{16} = a$

$y = -\frac{5}{16}(x+1)(x-7)$

$y = -\frac{5}{16}(x^2 - 6x - 7)$

b) $-4 = a(-3+1)(-3+5)$

$-4 = a(-2)(2)$

$-4 = -4a$ Opens up

$1 = a$

$y = (x+1)(x+5)$

$y = x^2 + 6x + 5$

c) $3 = a(0-3)(0-7)$

$3 = 21a$

$\frac{1}{7} = a$ Opens up

$y = \frac{1}{7}(x-3)(x-7)$

$y = \frac{1}{7}(x^2 - 10x + 21)$

$= \frac{1}{7}x^2 - \frac{10}{7}x + 3$

d) $-1 = a(-1+2)(-1-6)$

$-1 = a(1)(-7)$ Opens up

$\frac{1}{7} = a$

$y = \frac{1}{7}(x+2)(x-6)$

$= \frac{1}{7}(x^2 - 6x + 2x - 12)$

$= \frac{1}{7}(x^2 - 4x - 12)$

$= \frac{1}{7}x^2 - \frac{4}{7}x - \frac{12}{7}$

e) $7 = a(3+2)(3-8)$

$7 = a(5)(-5)$ opens down

$-\frac{7}{25} = a$

$y = -\frac{7}{25}(x+2)(x-8)$

$= -\frac{7}{25}(x^2 - 6x - 16)$

$= \frac{7}{25}x^2 + \frac{42}{25}x + \frac{112}{25}$

13. $2x^2+14x+20$ = Area

Check: $(2x+4)(x+5)$

$= 2x^2+10x+4x+20$

$= 2x^2+14x+20$ ☺

Check: $(2x+10)(x+2)$

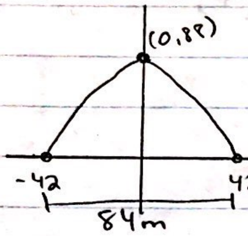
$= 2x^2+4x+10x+20$

$= 2x^2+14x+20$ ☺

∴ Either pair of dimensions work.

14. We know that the product will be quadratic because $(12x)(5x) = 60x^2$, so the equation will be second degree.

15.



$88 = a(0+42)(0-42)$

$88 = -1764a$

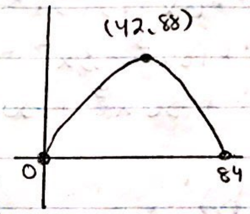
$\frac{-22}{441} = a$

$y = \frac{-22}{441}(x+42)(x-42)$

$= \frac{-22}{441}(x^2-42x+42x-1764)$

$= \frac{-22}{441}x^2 + 88$

OR



$88 = a(42-0)(42-84)$

$88 = -1764a$

$\frac{-22}{441} = a$

$y = \frac{-22}{441}(x-0)(x-84)$

$= \frac{-22x}{441}(x-84)$

$= \frac{-22}{441}x^2 + \frac{88}{21}$

16. Not always: $(x+1)(x-1)$

$= x^2-x+x-1$

$= x^2-1$ ← binomial

17a) $(x+3)^3$

$= (x+3)(x+3)(x+3)$

$= (x+3)(x^2+3x+3x+9)$

$= (x+3)(x^2+6x+9)$

$= x^3+6x^2+9x+3x^2+18x+27$

$= x^3+9x^2+27x+27$

17b) $(2x-2)^3$

$= (2x-2)(2x-2)(2x-2)$

$= (2x-2)(4x^2-4x-4x+4)$

$= (2x-2)(4x^2-8x+4)$

$= 8x^3-16x^2+8x-8x^2+16x-8$

$= 8x^3-24x^2+24x-8$

c) $(4x+2y)^3$

$= (4x+2y)(4x+2y)(4x+2y)$

$= (4x+2y)(16x^2+8xy+8xy+4y^2)$

$= (4x+2y)(16x^2+16xy+4y^2)$

$= 64x^3+64x^2y+32x^2y+32xy^2+16xy^2+8y^3$

$= 64x^3+96x^2y+48xy^2+8y^3$

17d) $[(x+2)(x-2)]^2$

$= (x^2-2x+2x-4)^2$

$= (x^2-4)^2$

$= (x^2-4)(x^2-4)$

$= x^4-4x^2-4x^2+16$

$= x^4-8x^2+16$

e) $(x+6)(x+3)(x-6)(x-3)$

$= (x+6)(x-6)(x+3)(x-3)$

$= (x^2-36)(x^2-9)$

$= x^4-9x^2-36x^2+324$

$= x^4-45x^2+324$

f) $(3x^2+6x-1)^2$

$= (3x^2+6x-1)(3x^2+6x-1)$

$= 9x^4+18x^3-3x^2+18x^3+36x^2-6x-3x^2-6x+1$

$= 9x^4+36x^3+30x^2-12x+1$

18a) $(a+b)^1$

$= a+b$

b) $(a+b)^2$

$= (a+b)(a+b)$

$= a^2+ab+ab+b^2$

$= a^2+2ab+b^2$

c) $(a+b)^3$

$= (a+b)(a+b)^2$

$= (a+b)(a^2+2ab+b^2)$

$= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$= a^3+3a^2b+3ab^2+b^3$

d) $(a+b)^4$

$= (a+b)^2(a+b)^2$

$= (a^2+2ab+b^2)(a^2+2ab+b^2)$

$= a^4+2a^3b+a^2b^2+2a^3b+4a^2b^2+2ab^3+a^2b^2+2ab^3+b^3$

$= a^4+4a^3b+6a^2b^2+4ab^3+b^3$

3.6 Exploring Quadratic + Exponential Graphs

Explore the Math

A.

# of Folds	# of Regions		
1	2		
2	4	2	
3	8	4	2
4	16	8	4
5	32	16	8
6	64	32	16
7	128	64	32

B. Find 2nd differences

The # of regions doubles with each fold.

∴ not quadratic

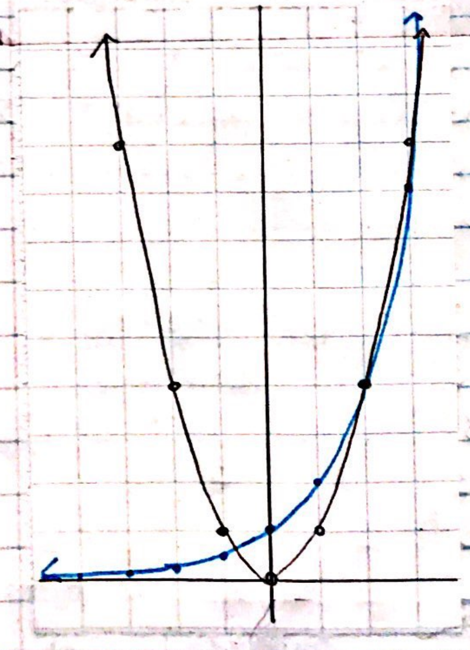
C. $y = 2^x$ works.

Let $x = 1$
$2^1 = 2$
$(1, 2)$

Let $x = 2$
$2^2 = 4$
$(2, 4)$

Let $x = 3$
$2^3 = 8$, etc.
$(3, 8)$

D. $y = x^2$, $y = 2^x$



E.
G.

x	$y = x^2$	$y = 2^x$
-5	25	$1/32$
-4	16	$1/16$
iv) $\boxed{-3}$	9	$\boxed{1/8}$
iii) $\boxed{-2}$	4	$\boxed{1/4}$
ii) $\boxed{-1}$	1	$\boxed{1/2}$
i) $\boxed{0}$	0	$\boxed{1}$
1	1	2
2	4	4
3	9	8
4	16	16
5	25	32

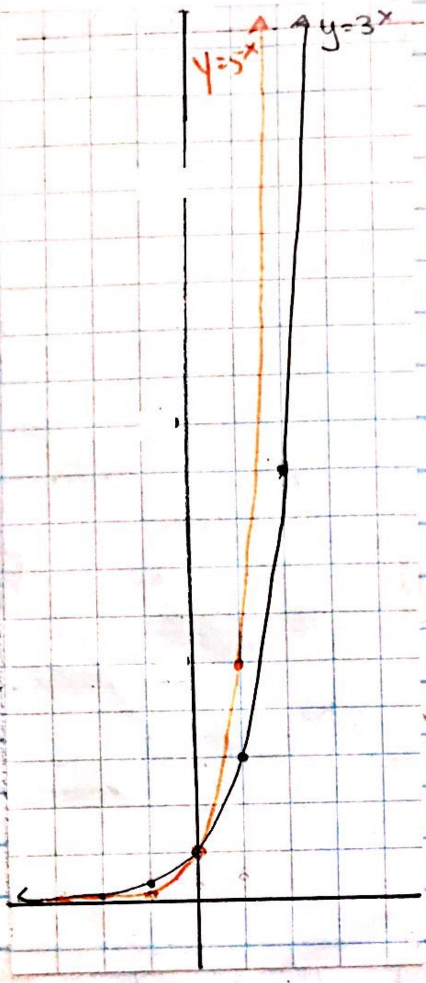
$y = 2^x$ is growing faster ($25 < 32$)

H. $(-1, \frac{1}{2^1})$, $(-2, \frac{1}{2^2})$, $(-3, \frac{1}{2^3})$

E. Both graphs are curves.
$y = x^2$ is a parabola
$y = 2^x$ is always ↑

I. i) $3^0 = 1$, $5^0 = 1$
ii) $3^{-1} = \frac{1}{3}$, $5^{-1} = \frac{1}{5}$
iii) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$, $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
iv) $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$, $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

J.



i) no symmetry

ii) y-int: (0, 1)

iii) The y values increase when x increases

iv) " y " decrease " x decreases

v) The y values also get large

vi) The y values approach zero

K. No! $y=2^x$ Let $x=-1000$

$$y=2^{-1000}$$
$$=\frac{1}{2^{1000}} \leftarrow \text{not zero!}$$

1 ÷ a # cannot equal zero!

L. i) $a^0=1$ ii) $a^{-1}=\left(\frac{1}{a}\right)^1$

iii) $a^{-2}=\left(\frac{1}{a}\right)^2$ iv) $a^{-n}=\left(\frac{1}{a}\right)^n$

M. In both $y=x^2$ and $y=2^x$, the y-values are always ⊕. For $y=x^2$, $(-)^2=+$, for $y=2^x$, a negative exponent flips the base, but the value remains positive.

Further Your Understanding

1a) $\frac{3^4}{3^4}$
$=3^{4-4}$
$=3^0$

b) $\frac{3^4}{3^4}$
$=\frac{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}}$
$=1$

c) $3^0=1$ because all factors cancel out.

d) $\frac{5^3}{5^3}$
$=5^{3-3}$
$=5^0$
$=1$

$\frac{5^3}{5^3}$
$=\frac{\cancel{5}\times\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}\times\cancel{5}}$
$=1$

2a) $\frac{3^3}{3^4}$
$=3^{3-4}$
$=3^{-1}$

b) $\frac{\cancel{3}\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}\times\cancel{3}\times 3}$
$=\frac{1}{3}$

c) $3^{-1}=\frac{1}{3}$

d) $\frac{5^2}{5^4}$
$=5^{-2}$
$=\frac{1}{25}$

$\frac{\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}\times 5\times 5}$
$=\frac{1}{25}$

e) $\frac{5^2}{5^5}$
$=5^{-3}$
$=\frac{1}{125}$

$\frac{\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}\times 5\times 5\times 5}$
$=\frac{1}{125}$

3a) 2^{-4}
$=(\frac{1}{2})^4$
$=\frac{1}{16}$

b) 4^{-1}
$=\frac{1}{4}$

c) 8^0
$=1$

d) 5^{-2}
$=(\frac{1}{5})^2$
$=\frac{1}{25}$

e) 3^{-4}
$=(\frac{1}{3})^4$
$=\frac{1}{81}$

f) 7^{-2}
$=(\frac{1}{7})^2$
$=\frac{1}{49}$

4a) $(-2)^{-5}$
$=(-\frac{1}{2})^5$
$=-\frac{1}{32}$

b) -4^{-2}
$=-(\frac{1}{4})^2$
$=-\frac{1}{16}$

c) -7^0
$=-1$

d) -5^{-1}
$=-(\frac{1}{5})$
$=-\frac{1}{5}$

e) $(-3)^{-2}$
$=(-\frac{1}{3})^2$
$=\frac{1}{9}$

f) $(-4)^{-3}$
$=(-\frac{1}{4})^3$
$=-\frac{1}{64}$

5a) $(\frac{1}{2})^2$
$=(\frac{1}{2})(\frac{1}{2})$
$=\frac{1}{4}$

b) $(\frac{1}{2})^{-2}$
$=2^2$
$=4$

c) $(\frac{2}{3})^3$
$=\frac{8}{27}$

d) $(\frac{2}{3})^{-3}$
$=(\frac{3}{2})^3$
$=\frac{27}{8}$

e) $-(\frac{3}{4})^{-2}$
$=-(\frac{4}{3})^2$
$=-\frac{16}{9}$

f) $(-\frac{3}{4})^{-2}$
$=(-\frac{4}{3})^2$
$=\frac{16}{9}$

6a) $2^n=\frac{1}{8}$
$2^n=\frac{1}{2^3}$
$2^n=2^{-3}$
$\therefore n=-3$

b) $4^n=64$
$4^n=4^3$
$\therefore n=3$

c) $5^n=1$
$5^n=5^0$
$\therefore n=0$

d) $n^{-3}=\frac{1}{27}$
$n^{-3}=3^{-3}$
$\therefore n=3$

e) $-3^n=-\frac{1}{9}$
$-3^n=-\frac{1}{3^2}$
$-3^n=-3^{-2}$
$\therefore n=-2$

f) $(-n)^4=16$
$n^4=16$
$n^4=2^4$
$\therefore n=2$

7. 5^{-2}
$=\frac{1}{25}$

10^{-2}
$=\frac{1}{100}$

$\therefore$ 5^{-2} is greater than 10^{-2}.

8. $(-1)^{-100}$
$=(-1)^{100}$
$=1$

$(-1)^{101}$
$=-1$

$\therefore$ $(-1)^{101}$ is less than $(-1)^{100}$

Hilary

Chapter 3 Review

1a) $y = 4x - 5$ → linear ($y = mx + b$)

b)

x	y		
-3	56		
		-21	
-2	35		4
		-17	
-1	18		4
		-13	
0	5		4
		-9	
1	-4		4
		-5	
2	-9		4
		-1	
3	-10		

Quadratic → equal 2nd diff.

c) $y = 2x(x-5)$

$= 2x^2 - 10x$

Quadratic → second degree

d) Quadratic → parabola

2. a → ⊕ opens up, ⊖ opens down

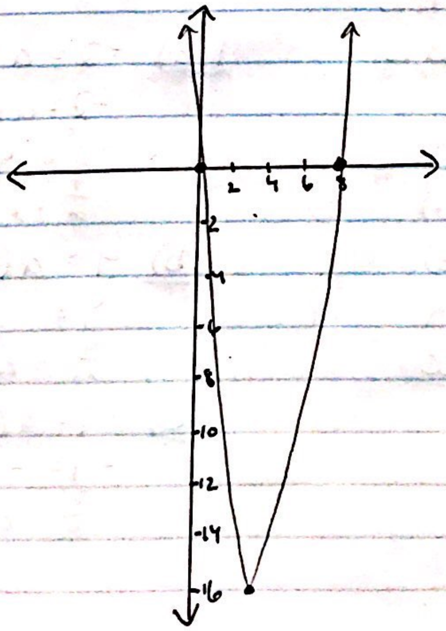
→ large #, skinnier; fraction, fatter

c → y-intercept

b → moves left/right,

3a)

x	$y = x^2 - 8x$
0	0
1	-7
2	-12
3	-15
4	-16
5	-15
6	-12
7	-7
8	0



i) $x = 4$

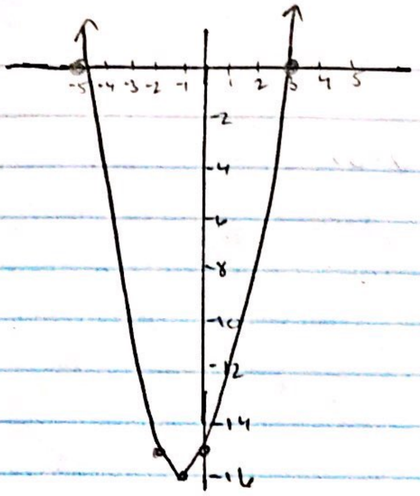
ii) $(4, -16)$

iii) $(0, 0)$

iv) $(0, 0) + (8, 0)$

3b)

x	y
-2	-15
-1	-16
0	-15
1	-12
2	-7
3	0



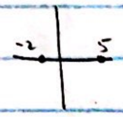
i) $x = -1$

ii) $(-1, -16)$

iii) $(0, -15)$

iv) $(3, 0)$ and $(-5, 0)$

5.



a) opens down, so it is a maximum

b) positive → above the x-axis

c) $x = \frac{-2+5}{2}$

$x = \frac{3}{2}$

7a) i) $(0, 0) + (18, 0)$

ii) $x = 9$

iii) $(9, 81)$

b) i) $(0, 0) + (-\frac{5}{2}, 0)$

ii) $x = {-5/4}$

iii) $(-\frac{5}{4}, \frac{-25}{8})$

8. 'a' tells us if it opens up or down, and how wide the parabola is.

9. $y = -6x^2 + 42x - 60$

$y = -6(x^2 - 7x + 10)$

$= -6(x-5)(x-2)$

a) They break even at 5000 and 2000 sales.

b) $x = \frac{5+2}{2} = 3.5$

∴ They need to sell 3500 boards.

10a) $y = a(x-r)(x-s)$

$-28 = a(0+2)(0-7)$

$-28 = (-14)a$

$2 = a$

$y = 2(x+2)(x-7)$

b) $x = \frac{-2+7}{2}$

$= \frac{5}{2}$

$y = 2(\frac{5}{2}+2)(\frac{5}{2}-7)$

$= 2(\frac{9}{2})(-\frac{9}{2})$

$= -\frac{81}{2}$

Vertex: $(\frac{5}{2}, -\frac{81}{2})$

11a) $-2 = a(7-5)(7-9)$

$-2 = a(2)(-2)$

$-2 = -4a$

$\frac{1}{2} = a$

$y = \frac{1}{2}(x-5)(x-9)$

b) $4 = a(2+3)(2-7)$

$4 = -25a$

$-\frac{4}{25} = a$

$y = \frac{-4}{25}(x+3)(x-7)$

c) $-9 = a(0+6)(0-2)$

$-9 = -12a$

$\frac{3}{4} = a$

$y = \frac{3}{4}(x+6)(x-2)$

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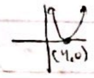
11d) 

$8 = a(0-4)(0-4)$

$8 = 16a$

$\frac{1}{2} = a$

$y = \frac{1}{2}(x-4)^2$

e) $20 = a(2+3)(2-3)$

$20 = -5a$

$-4 = a$

$y = -4(x+3)(x-3)$

13a) $(x+3)(2x-3)$

$= 2x^2 + 3x - 9$

b) $(5x-6)(3x-4)$

$= 15x^2 - 38x + 24$

12. Let x be the number of \$0.10 price increases.

R = price × quantity

$R = (1+0.1x)(12000-400x)$

Break even: $1+0.1x = 0$ $12000-400x = 0$

$0.1x = -1$ $-400x = -12000$

$x = -10$ $x = 30$

Vertex: $x = \frac{-10+30}{2}$

$= 10 \swarrow$ Increase price 10x0.1, so \$1.

$\therefore$ They should charge \$2.00.

14a) $(x+5)(x+4)$

$= x^2 + 9x + 20$

b) $(x-2)(x-5)$

$= x^2 - 7x + 10$

c) $(2x-3)(2x+3)$

$= 4x^2 + 6x - 6x - 9$

$= 4x^2 - 9$

d) $(4x+5)(3x-2)$

$= 12x^2 - 8x + 15x - 10$

$= 12x^2 + 7x - 10$

e) $(4x-2y)(5x+3y)$

$= 20x^2 + 12xy - 10xy - 6y^2$

$= 20x^2 + 2xy - 6y^2$

f) $(6x-2)(5x+7)$

$= 30x^2 + 42x - 10x - 14$

$= 30x^2 + 32x - 14$

15a) $(2x+6)^2$

$= (2x+6)(2x+6)$

$= 4x^2 + 12x + 12x + 36$

$= 4x^2 + 24x + 36$

b) $-2(-2x+5)(3x+4)$

$= -2[-6x^2 - 8x + 15x + 20]$

$= 12x^2 - 14x - 40$

c) $2x(4x-y)(4x+y)$

$= 2x(16x^2 + 4xy - 4xy - y^2)$

$= 32x^3 - 2xy^2$

16. $8 = a(0+4)(0-8)$

$8 = -32a$

$-\frac{1}{4} = a$

$y = -\frac{1}{4}(x+4)(x-8)$

$= -\frac{1}{4}(x^2 - 8x + 4x - 32)$

$= -\frac{1}{4}x^2 + x + 8$

19a) 2^{-3}
$= \left(\frac{1}{2}\right)^3$
$= \frac{1}{8}$

b) -5^{-1}
$= -\frac{1}{5}$

c) $\left(\frac{2}{5}\right)^{-2}$
$= \left(\frac{5}{2}\right)^2$
$= \frac{25}{4}$

d) $(-9)^0$
$= 1$

e) 4^{-3}
$= \left(\frac{1}{4}\right)^3$
$= \frac{1}{64}$

f) $-\left(\frac{1}{6}\right)^{-2}$
$= -(6)^2$
$= -36$

20. $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$
3^{-2}
$= \left(\frac{1}{3}\right)^2$ $\quad \frac{1}{9} > \frac{1}{16}$
$= \frac{1}{9}$

21. $x^2 > 2^x$ when $x < 0$.

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Chapter 3 Self-Test

1. zeros: (-6, 0) and (2, 0)
vertex: (-2, -4)
a of s: x = -2

2a) $x = \frac{-9+19}{2}$

$x = 5$

b) $y = a(x-r)(x-s)$
$-28 = a(5+9)(5-19)$
$-28 = 196a$
$-\frac{1}{7} = a$
$y = -\frac{1}{7}(x+9)(x-19)$

c) $y = -\frac{1}{7}(x^2-19x+9x-171)$
$= -\frac{1}{7}(x^2-10x-171)$
$= -\frac{1}{7}x^2 + \frac{10}{7}x + \frac{171}{7}$

3a)

x	y		
-1	1		
		1	
0	2		-6
		-5	
1	-3		-6
		-11	
2	-14		-6
		-17	
3	-31		

∴ This is quadratic

b)

x	y		
0	-4		
		1	
1	-3		2
		3	
2	0		2
		5	
3	5		2
		7	
4	12		

∴ This is quadratic

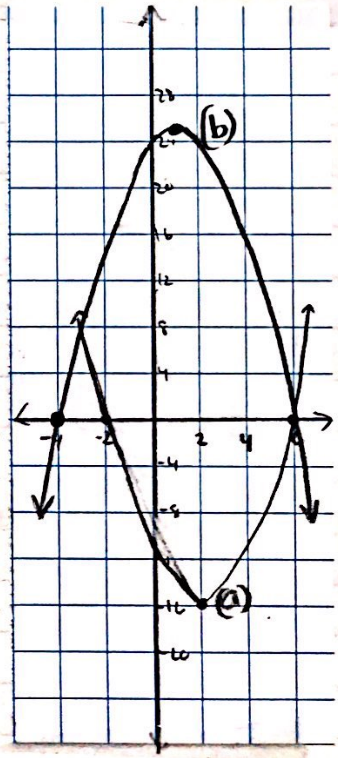
4a) $y = (x-6)(x+2)$
a of s: $x = \frac{6-2}{2}$
$x = 2$
$y = (2-6)(2+2)$
$= -16$
Vertex (2, -16)

b) $y = -(x-6)(x+4)$
a of s: $x = \frac{6-4}{2}$
$x = 1$
$y = -(1-6)(1+4)$
$= 25$
Vertex: (1, 25)

5. $P = 14t^2 + 820t + 42000$

a) $t = 10$
$P = 14(10)^2 + 820(10) + 42,000$
$= 51600$
∴ The population is 51600 in 2018

b) $30000 = 14t^2 + 820t + 42000$
$14t^2 + 820t + 12000 = 0$



6a) $(2x-3)(5x+2)$
$= 2x(5x+2) - 3(5x+2)$
$= 10x^2 + 4x - 15x - 6$
$= 10x^2 - 11x - 6$

b) $(3x-4y)(5x+2y)$
$= 3x(5x+2y) - 4y(5x+2y)$
$= 15x^2 + 6xy - 20xy - 8y^2$
$= 15x^2 - 14xy - 8y^2$

c) $-5(x-4)^2$
$= -5(x-4)(x-4)$
$= -5[x(x-4) - 4(x-4)]$
$= -5(x^2 - 4x - 4x + 16)$
$= -5(x^2 - 8x + 16)$
$= -5x^2 + 40x - 80$

7a) 16m
b) 8 seconds
c) Very close to one
f) a little higher than 88m

9a) 7^{-2}
$= \left(\frac{1}{7}\right)^2$
$= \frac{1}{49}$

b) -3^0
$= -1$

c) $-\left(\frac{2}{3}\right)^{-4}$
$= -\left(\frac{3}{2}\right)^4$
$= -\frac{81}{16}$

d) -5^{-3}
$= -\left(\frac{1}{5}\right)^3$
$= -\frac{1}{125}$