Date: ______________ 

Work through this at your own pace - you have three days!

## 7.4 to 7.6 The Primary Trigonometric Ratios: Working with Right Triangles

### 1) Vocabulary

trigonometry - the branch of mathematics concerned with the properties of triangles and calculations based on these properties

ratio - a comparison of two quantities measured in the same units

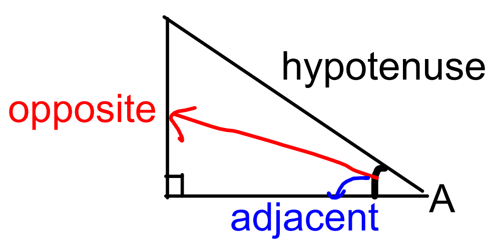
primary trigonometric ratios - the basic ratios of trigonometry; ratios of side lengths that are true for a given angle

The primary trigonometric ratios are called **sine, cosine, and tangent.** On your calculator, they are labelled as sin, cos, and tan. If you cannot find these buttons, but have a scientific calculator, contact me and I will help you.

**We ALWAYS have to attach sin, cos, or tan to an angle! They do not do anything on their own.**

### 2) The Primary Trig Ratios

When you have a right triangle, you can identify ratios for each angle based on the location of the sides. The sides need to be named opposite, adjacent, and hypotenuse before we can start to make ratios.

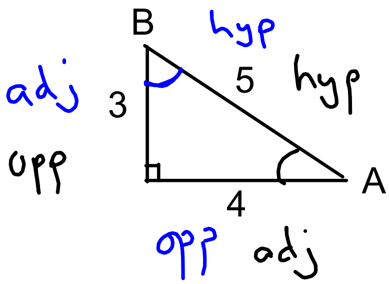



Once we have identified our sides, we can compare them to one another.

$$\boxed{\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan A = \frac{\text{opposite}}{\text{adjacent}}}$$

These are ratios comparing the side lengths. Every $30^{\circ}$ angle will have the same values of sin, cos, and tan, regardless of the size of the triangle itself, as the sides will increase proportionally.

Let's try writing the primary trig ratios for angle A in the triangle shown below.




$\sin A = \frac{O}{H}$ $\cos A = \frac{A}{H}$ $\tan A = \frac{O}{A}$

$\sin A = \frac{3}{5}$ $\cos A = \frac{4}{5}$ $\tan A = \frac{3}{4}$

Now let's write the primary trig ratios for angle B in the same triangle. What do you notice?

$\sin B = \frac{4}{3}$ $\cos B = \frac{3}{5}$

$\tan B = \frac{4}{3}$

$\sin A = \cos B$

$\cos A = \sin B$

tan B is the reciprocal of tan A.

**NEVER USE THE 90° ANGLE TO SET UP PRIMARY TRIG RATIOS!!!**

A shortcut to remember the primary trig ratios is to remember the acronym:

**SOH** **CAH** **TOA**

SOH: sine, opposite, hypotenuse

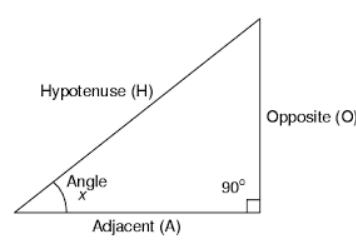
CAH: cosine, adjacent, hypotenuse

TOA: tangent, opposite, adjacent

As long as you label the sides appropriately and remember this acronym, you should be able to write primary trig ratios for any angle in a right triangle!

You can also still use the Pythagorean Theorem to find the length of the third side of a right triangle once you know the other two!

**We are going to try Practice Problem #1 now.**



3) <u>Using Your Calculator to Find Values for Trig Expressions</u>

a) <u>Finding a Ratio Given an Angle</u>

ex/ Determine the value of sin 35° to four decimal places.

$\sin 35° = 0.5736 \leftarrow opp \div hyp$

Calculating values like this with your calculator is pretty straight forward once you figure out how your calculator works.

If you have a two line display, you will type it in as it appears:

$\boxed{\sin}\ \boxed{3}\ \boxed{5}\ \boxed{=}$

If you have a single line display, you will probably need to type it in with the angle first, then the trig ratio. $\boxed{3}\ \boxed{5}\ \boxed{\sin}\ \boxed{=}$

This solution just represents the ratio of the opposite side to the hypotenuse for any right triangle with a 35° angle in it.

b) <u>Finding an Angle Given a Ratio</u>

Your calculator also has **inverse** trig ratios ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$). These are located on the same keys as your primary trig ratios, but you need to hit shift or second to get to them. **If you have to find an angle you MUST use these keys!!**

Deg

Rad

Grad

ex/ Determine the measure of x to the nearest tenth of a degree.

sin x = 0.4365

$x = \sin^{-1}(0.4365)$

$= 25.9$

cos x = 0.5673

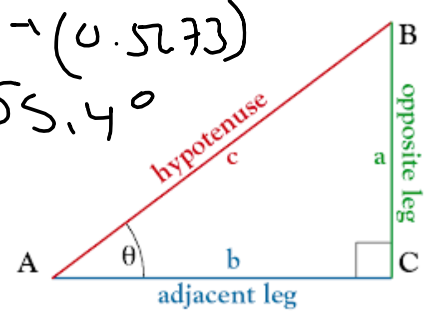
$x = \cos^{-1}(0.5673)$

$= 55.4°$

tan x = 1.233

$x = \tan^{-1}(1.233)$

$= 51.0°$

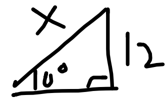



Now we can do Practice Problem #2

## 4) Solving Equations Involving Trig Functions

The rules for solving equations stay the same. You use inverse operations to undo everything that has been done to your unknown. We just need to remember that we use inverse trig functions to "get rid of" trig ratios.

ex/ Solve for x to one decimal place.




a) $(14)\tan 47^\circ = \frac{x}{14}(14)$   opp / adj

$14\tan 47^\circ = x$

$15.0 = x$

b) $(x)\sin 10^\circ = \frac{12}{x}(x)$

$\frac{x\sin 10^\circ}{\sin 10^\circ} = \frac{12}{\sin 10^\circ}$

$x = 69.1$

It is very important to remember that if you are solving for a **side** (x is in the ratio), you are using **regular buttons** on your calculator, but if you are solving for an **angle** (x is with sin, cos, or tan), you are using **inverse trig buttons**.

## 5) Using Trig to Find Unknown Values in Right Triangles

Now that we know the primary trig ratios, and how to use our calculators to solve them we can apply our knowledge to triangles.

In order to solve for unknown values in triangles, you must:

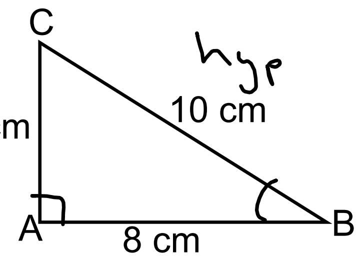
- Draw a diagram if one is not given;
- Accurately label your sides as opposite, adjacent and hypotenuse (relative to the angle that you are interested in);
- Determine which trig ratio will allow you to solve for your unknown value (you must know two of three variables);
- Sub your values in appropriately and solve accurately.

Remember the things that you already know about triangles!

- The angles inside a triangle always add to 180°;
- The Pythagorean theorem will allow you to find the length of the third side of a right triangle if you know the other two!

Practice Problems

1) For the triangle shown, find sin B, cos B, and tan B. Express your answer as a ratio in lowest terms.




hyp

opp

adj

$\sin B = \frac{O}{H} = \frac{6}{10} = \frac{3}{5}$

$\cos B = \frac{A}{H} = \frac{8}{10} = \frac{4}{5}$

$\tan B = \frac{O}{A} = \frac{6}{8} = \frac{3}{4}$

2) Calculate the measure of angle B in 1).

$\sin B = \frac{3}{5}$

$B = \sin^{-1}\left(\frac{3}{5}\right)$

$= 36.9°$

$\cos B = \frac{4}{5}$

$B = \cos^{-1}\left(\frac{4}{5}\right)$

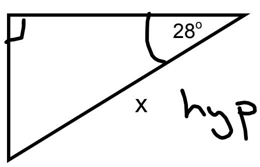
$= 36.9°$

Pick one ratio + solve!

3) For each of the following triangles, find the value of x to one decimal place.

SOH

a)




opp

hyp

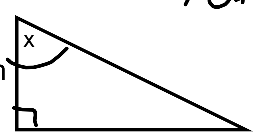
$\sin 28° = \frac{3}{x}$

$x = \frac{3}{\sin 28°}$

$x = 6.4\text{ cm}$

b)

TOA




adj

opp

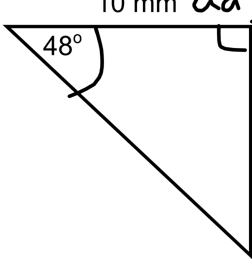
$\tan x = \frac{15}{7}$

$x = \tan^{-1}\left(\frac{15}{7}\right)$

$x = 65°$

c)

adj




opp

TOA

$10(\tan 48°) = \left(\frac{x}{10}\right)10$

$10 \tan 48° = x$

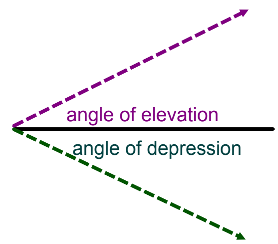
$11.1\text{mm} = x$

6) Solving Right Triangle Problems (This is Lesson #2)

To solve word problems involving triangles, you should:
- draw a diagram.
- write a "let" statement, clearly defining what you are trying to find.
- label your diagram appropriately with the given information and your unknown.
- choose the appropriate trig ratio and solve.
- write a "therefore" statement.

angle of elevation - an angle formed between the horizontal and a line that rises above it

angle of depression - an angle formed between the horizontal and a line that falls below it



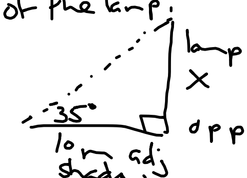

***THESE MUST BE MEASURED FROM THE HORIZONTAL!!***

Things to Watch for:

- Sometimes you will need to use more than one right triangle to get an answer.
- You must be able to draw diagrams involving angles of elevation (inclination) and depression!
- You will sometimes be asked to use logic and reasoning to explain your answer.

Practice Problems Part 2

1) A street lamp casts a shadow that is 10 m long. The sun's rays meet the ground at an angle of 35°. Determine the height of the street lamp the nearest tenth of a meter. Let x be the height of the lamp.




TOA

$10(\tan 35°) = \left(\frac{x}{10}\right)10$

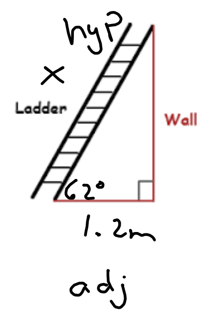
$10\tan 35° = x$

$7.0 = x$

∴ the lamp is 7m tall.

2) A ladder forms an angle of 62° with the ground. If the base of the ladder is 1.2 m from the wall, how long is the ladder?

Let x be the length of the ladder.




CAH

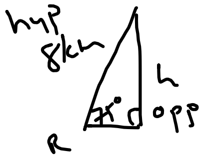
$\cos 62° = \frac{1.2}{x}$

$x = \frac{1.2}{\cos 62°}$

$x = 2.6\text{m}$

∴ The ladder is 2.6m long.

3) A rocket is launched at an angle of $75^{\circ}$ to the ground and travels in a straight line. What is its altitude when it has travelled for 8 km?




Let h be the altitude. SOH

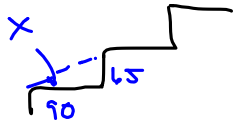
$\sin 75 = \dfrac{h}{8}$

$8\sin 75^{\circ} = h$

7.7km = h

∴ The altitude is 7.7km.

4) If the maximum slope for a set of stairs is 65 cm of rise for every 90 cm of run, what is the maximum angle of elevation for a set of stairs?



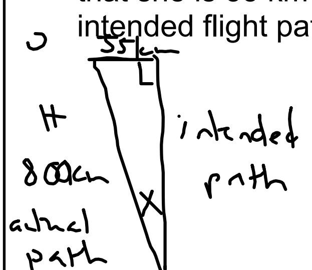

Let x be the angle of elevation. TOA

$\tan x = \dfrac{65}{90}$

$x = 35.8^{\circ}$

∴ The max angle of elevation is 35.8°.

5) The pilot of an airplane is flying at 400 km/h. After two hours, she notices that she is 55 km west of her intended course. At what angle to her intended flight path was she flying? SOH




Let x be the angle to her intended path.

$\sin x = \dfrac{55}{800}$

$x = 3.9^{\circ}$

∴ She is 3.9° from her intended path.

6) Gabby is sitting on the roof of an apartment building. She can see the top of the building across the road if she looks up through an angle of $30^{\circ}$. She can see the base of the building if she looks down through an angle of $42^{\circ}$. If the distance between the buildings is 10 m, how tall is the building across the road?

Let x be the height of the building.

$x = x_1 + x_2$

$\tan 30^{\circ} = \dfrac{x_1}{10}$

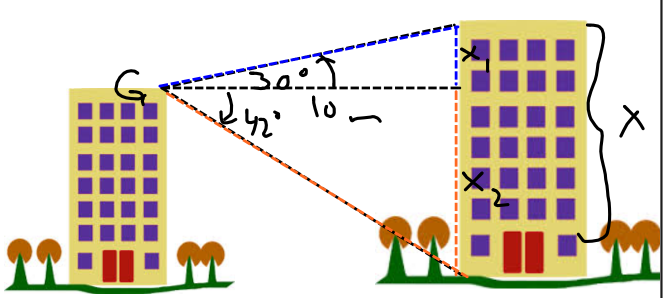
$10\tan 30^{\circ} = x_1$

$5.8\text{m} = x_1$

$\tan 42^{\circ} = \dfrac{x_2}{10}$

$10\tan 42^{\circ} = x_2$

$9.0\text{m} = x_2$




∴ The building is 14.8m tall.