Date: ____________________

6.4 The Quadratic Formula

We can find roots of a quadratic equation that cannot be factored directly using the **quadratic formula**.

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$ax^2+bx+c=0$

Where a, b, and c are the coefficients from a quadratic equation in **standard form**.

This formula allows us to find roots (or solve equations) without having to partial factor and write in vertex form.

Warnings About the Formula:

- Sign errors are common. Use brackets to substitute!
- Factoring is often more efficient, so don't forget how to do it.
- You should get two answers! Don't forget to add and subtract the root.
- BEDMAS errors are common. Be careful and make sure you know how to work your calculator!!

Example: Solve $5x^2 - 4x - 3 = 0$ using the quadratic formula. Round your solutions to two decimal places.

$a=5$

$b=-4$

$c=-3$

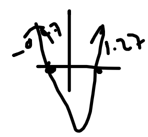
$$x = \frac{4 \pm \sqrt{(-4)^2-4(5)(-3)}}{2(5)}$$

$$x = \frac{4 \pm \sqrt{16+60}}{10}$$

$$x = \frac{4 \pm \sqrt{76}}{10}$$

$$x = \frac{4+\sqrt{76}}{10} = 1.27$$

$$x = \frac{4-\sqrt{76}}{10} = -0.47$$



Now solve $x^2 - x - 6 = 0$ by factoring and by using the formula. Which was more efficient?

Formula

$a=1 \quad b=-1 \quad c=-6$

$$x = \frac{1 \pm \sqrt{(-1)^2-4(1)(-6)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{25}}{2}$$

$x = \dfrac{1+5}{2}$

$\boxed{x=3}$

$x = \dfrac{1-5}{2}$

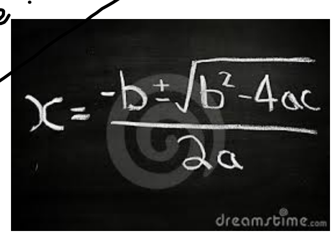
$\boxed{x=-2}$

Factor

$(x-3)(x+2)=0$

$x=3 \quad x=-2$

Same!



When you are solving quadratic equations, always choose the most appropriate/efficient method.

Form of Equation

- Ch. 4/3 → **Factored**: $a(x - r)(x - s) = 0$ → Set each factor equal to zero
- Ch. 5 → **Vertex**: $a(x - h)^2 + k = 0$ → Rearrange!
- **Standard**: $ax^2 + bx + c = 0$ → Factor (Ch. 4) → Set each factor equal to zero; or → Quadratic Formula (Ch. 6.)

More Practice:

1) Find the roots of $y = 2x^2 - 6x + 2$ when $y = -1.5$.

(0.79, −1.5) and (2.21, −1.5)

$2x^2 - 6x + 2 = -1.5$

$2x^2 - 6x + 3.5 = 0$

QF (decimal !!)

$a = 2$ $\quad c = 3.5$

$b = -6$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(3.5)}}{2(2)}$

$= \frac{6 \pm \sqrt{8}}{4}$

$x = \frac{6 + \sqrt{8}}{4} = 2.21$ $\qquad x = \frac{6 - \sqrt{8}}{4} = 0.79$

2) Determine the roots of $(x + 4)^2 = 2(x + 5)$. Round to two decimal places.

$(x + 4)(x + 4) = 2x + 10$

$x^2 + 4x + 4x + 16 = 2x + 10$

$x^2 + 6x + 6 = 0$

QF $\quad a = 1$

$b = 6$

$c = 6$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2(1)}$

$= \frac{-6 \pm \sqrt{36 - 24}}{2}$

$x = \frac{-6 + \sqrt{12}}{2} = -1.27$ $\qquad x = \frac{-6 - \sqrt{12}}{2} = -4.73$

3) Solve $-2(x + 4)^2 + 12 = 0$. Report your answers as exact values.

Rearrange.

Leave square roots, fractions

$$-2(x+4)^2+12=0$$

$$\frac{-2(x+4)^2}{-2} = \frac{-12}{-2}$$

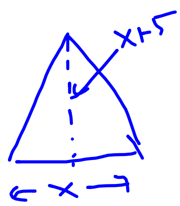
$$\sqrt{(x+4)^2} = \pm\sqrt{6}$$

$$x+4 = \pm\sqrt{6}$$

$$x = -4 \pm \sqrt{6}$$

answers as exact values.

4) A triangle's height is 5 cm longer than its base. The area of the triangle is 53 cm^2. Determine the dimensions of the triangle.



$A = \frac{bh}{2}$

Let x is the length of the base.

$$2\left[\frac{x(x+5)}{2}\right] = 2(53)$$

$$x^2+5x=106$$

$$x^2+5x-106=0$$

$a=1$, $b=5$, $c=-106$

$$x = \frac{-5 \pm \sqrt{(5)^2-4(1)(-106)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{449}}{2}$$

$$\cancel{x = \frac{-5 - \sqrt{449}}{2}} \qquad x = \frac{-5+\sqrt{449}}{2}$$

$$x = 8.09$$

∴ The base is 8.09 cm + the height is 13.09 cm.

5) A ball is thrown from a building. Its path is modelled by $h=-5t^2+8t+10$, where h is height in m and t is time in seconds. Determine the length of time that the ball spends above 12 m.

$a=5$

$b=-8$

$c=2$

$$h=-5t^2+8t+10$$

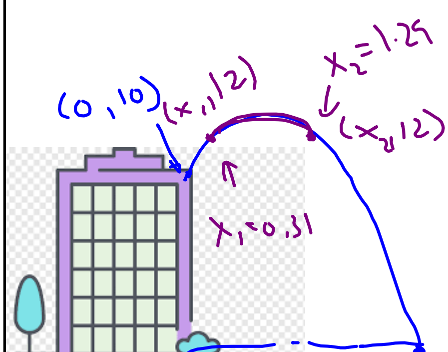
$$12=-5t^2+8t+10$$

$$5t^2-8t+2=0$$

$$x = \frac{8 \pm \sqrt{(-8)^2-4(5)(2)}}{2(5)}$$

$$= \frac{8 \pm \sqrt{24}}{10}$$

$$x = \frac{8+\sqrt{24}}{10} = 1.29\text{ s} \qquad x = \frac{8-\sqrt{24}}{10} = 0.31\text{ s}$$



$t = 1.29 - 0.31$

$= 0.98$ s

∴ The ball is above 12 m for 0.98 s.