Date: ____________________

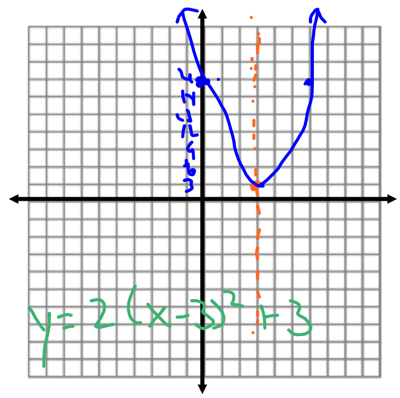
5.6 Connecting Standard and Vertex Form

Changing Standard Form to Vertex Form when Factoring is not Possible

When a parabola does not have zeros, you cannot write an equation in factored form. If the parabola does not have whole number zeros it is also difficult to write equations in factored form. When these situations arise, we can use a method called **partial factoring**.

To Partial Factor:

- group the first two terms in the equation and common factor them.
- set each partial factor equal to zero to get the x - coordinates of two points that are the same distance from the axis of symmetry.
- add and divide by two to get 'h'.
- sub into the original equation to get 'k'.



a

Example: Write $y = \boxed{2x^2 - 12x} + 21$ in vertex form by partial factoring.

$y = 2x(x-6) + 21$

$2x = 0$ $\quad$ $x - 6 = 0$

$x = 0$ $\quad$ $x = 6$

Points: $(0, 21)$, $(6, 21)$

A of S: $x = \frac{0+6}{2}$

$x = 3$

$y = 2(3)^2 - 12(3) + 21$

$= 18 - 36 + 21$

$y = 3$

Shortcut to Partial Factor:

Partial factor $y = ax^2 + bx + c$ so that we can come up with a general approach to finding 'h'.

① Partial Factor

$y = ax^2 + bx + c$

$y = x(ax + b) + c$

$x = 0$ $\quad$ $ax + b = 0$

$ax = -b$

$x = \frac{-b}{a}$

② Find a of s

$x = \frac{0 + \frac{-b}{a}}{2}$

$x = \frac{-b}{a} \div 2$

$= \frac{-b}{a} \times \frac{1}{2}$

$x = \frac{-b}{2a}$

Gives you the x-coord of your vertex from standard form.

Factored Form	Expand & Simplify → / ← Factor	Standard Form	Partial Factor $x = \frac{-b}{2a}$ → / ← Expand & Simplify	Vertex Form
$y = a(x - r)(x - s)$		$y = ax^2 + bx + c$		$y = a(x - h)^2 + k$

Practice Problems

1) A parabola is modelled by the relationship $y = \overset{a}{4}x^2 - \overset{b}{5}x + \overset{c}{7}$. Write the relationship in vertex form and ① identify the axis of symmetry and the ② optimal value for the parabola.

$b = -5$

① A of S

$x = \frac{-b}{2a}$

$x = \frac{5}{2(4)}$

$x = 5/8$ (h)

② Optimal value

$y = 4\left(\frac{5}{8}\right)^2 - 5\left(\frac{5}{8}\right) + 7$

$= 4\left(\frac{25}{64}\right) - \frac{25}{8} + 7$

$= \frac{25}{16} - \frac{50}{16} + 7$

$= \frac{87}{16}$ (k)

③ Equation

$y = a(x-h)^2 + k$

$y = 4\left(x - \frac{5}{8}\right)^2 + \frac{87}{16}$

2) Graph the relationship $y = -2x^2 - 6x + 1$ by first writing the relation in vertex form. State the transformations that have been applied to the graph of $y = x^2$ to produce your graph. (Connection alert! This does relate to the first half of the chapter :))

① A of S

$x = \frac{-b}{2a}$

$= \frac{6}{2(-2)}$

$= -\frac{3}{2}$

② y-coord

$y = -2\left(-\frac{3}{2}\right)^2 - 6\left(-\frac{3}{2}\right) + 1$

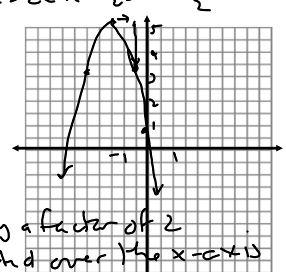
$= -2\left(\frac{9}{4}\right) + 9 + 1$

$= -\frac{9}{2} + 10$

$= \frac{11}{2}$

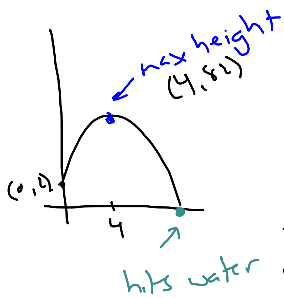
Vertex: $\left(-\frac{3}{2}, \frac{11}{2}\right)$

③ $y = -2\left(x + \frac{3}{2}\right)^2 + \frac{11}{2}$



④ v.s. by a factor of 2
reflected over the x-axis
shifted $\frac{3}{2}$ units left and $\frac{11}{2}$ units up.

3) The height of a rocket launched from a barge is modelled by $h = -5t^2 + 40t + 2$, where h is the height in meters above the water and t is time in seconds after it is launched. Determine the maximum height of the rocket and the time that the rocket will hit the water.



① Max height

$t = \frac{-40}{-10}$

$= 4$

$h = -5(4)^2 + 40(4) + 2$

$= -80 + 160 + 2$

$= 82$ m

∴ The max height is 82 m.

② Water entry

$h = -5(t-4)^2 + 82$

Let h = 0: $-5(t-4)^2 + 82 = 0$

$\frac{-5(t-4)^2}{-5} = \frac{-82}{-5}$

$\sqrt{(t-4)^2} = \sqrt{16.4}$

$t - 4 = 4.05$

$t = 8.05$ seconds.

∴ The rocket hits the water after 8.05 seconds.