Date: _____

5.4 Quadratic Models Using Vertex Form

Vertex Form: $y = a(x - h)^2 + k$
(x, y) → Any point on the graph
(h, k) → The vertex

You can write equations in vertex form if you are given:

- a list of transformations. ✓
- the vertex and a value for a.
- the vertex and any other point.
- the zeros and a point.

* Equations have an x + y in them!!

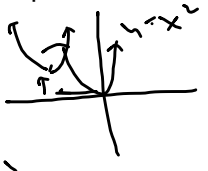
For the first two options, you simply need to be able to sub in the values in the appropriate places.

Ex. 1/ The graph of $y = x^2$ has been stretched vertically by a factor of 8, and shifted 5 units to the left and two units up. Write the equation of the new relation. How many zeros will it have?

$a = 8$ $\quad k = +2$
$h = -5$

$y = a(x-h)^2 + k$
$y = 8(x+5)^2 + 2$

no zeros



Ex. 2/ A quadratic relation with an equation of the form $y = a(x - h)^2 + k$ has been vertically compressed by a factor of 0.25. Its vertex is at (-4, 7). Write the relation.

$a = 0.25$ $\quad h$ $\quad k$

$y = 0.25(x+4)^2 + 7$

The other two situations require you to apply skills that you already know.

When you are given the vertex and a point, you must solve for 'a' before you can write the relation in vertex form.

Ex. 3/ A parabola has a vertex at (2, -3) and passes through the point (3, 8). Determine a relation in vertex form to model the parabola.

$h = 2$ $\quad x = 3$
$k = -3$ $\quad y = 8$

$y = a(x-h)^2 + k$
$8 = a(3-2)^2 - 3$
$8 = a(1)^2 - 3$
$8 + 3 = a - 3 + 3$
$11 = a$ → $y = 11(x-2)^2 - 3$ equation!

B
C

If you are given the zeros and a point, you must find the vertex, and then sub in the vertex and your point to solve for 'a'.

Ex. 4/ A parabola has zeros at (-1, 0) and (5, 0), and passes through the point (3, -8). Determine a relation in vertex form to represent the parabola.

AOFS

$x = \frac{-1+5}{2}$

$x = 2$

$\therefore h = 2$

① Factored form

$y = a(x+1)(x-5)$

$-8 = a(3+1)(3-5)$

$-8 = a(4)(-2)$

$-8 = -8a$

$1 = a$

② Find y-coord of vertex

$y = (x+1)(x-5)$

$y = (2+1)(2-5)$

$y = (3)(-3)$

$y = -9$ $\therefore k = -9$

③ Vertex form: $y = 1(x-2)^2 - 9$

To write a relation in vertex form in standard form, expand and simplify.

Ex. 5/ Write the relation in Ex. 4 in standard form.

$y = (x-2)^2 - 9$

$= (x-2)(x-2) - 9$

$= x^2 - 2x - 2x + 4 - 9$

$= x^2 - 4x - 5$ ← standard form

Ex. 6/ The Warm Bread Bakery wants to determine a quadratic relation to model the daily profit from bread sales. They know that they reach a maximum profit of $400 on days when they sell their bread at a price of $1.75 per loaf. They also know that when they sell their bread for $0.75 per loaf they make a profit of $300. Can you help them write a relation to model this?

Vertex

Vertex: (1.75, 400)
h k

Point: (0.75, 300)
x y

$y = a(x-h)^2 + k$

$300 = a(0.75-1.75)^2 + 400$

$300 = a(-1)^2 + 400$

$300 = a + 400$

$-100 = a$

Answer!

$y = -100(x-1.75)^2 + 400$