Date: ____________________

(MOST IMPORTANT LESSON OF THE YEAR!)

4.4 Factoring Quadratics: $ax^2 + bx + c$

expand & simplify

$$
\begin{aligned}
&2x^2 + 9x + 10 \\
&= 2x^2 + 5x + 4x + 10 \\
&= (2x^2 + 5x) + (4x + 10) \\
&= x(2x + 5) + 2(2x + 5) \\
&= (x + 2)(2x + 5)
\end{aligned}
$$

factor

☆ ALWAYS LOOK FOR A COMMON FACTOR FIRST!!

Factoring Trinomials of the Form $y = ax^2 + bx + c$ Using Decomposition

Decomposition is the opposite process to our new expanded distributive property from Chapter 3. We can apply our understanding of that and our new factoring by grouping skills to factor quadratic relations in standard form!

$y = ax^2 + bx + c$

Steps:

1) Multiply a and c.

2) **Find two numbers that multiply to give ac and add to give b.**
   ( )( ) = ac
   ( ) + ( ) = b

3) Split the bx term in to two terms using your values from step 2. This will leave you with four terms.

4) Common factor this expression by grouping.

5) If you have done this right, the binomial factors will always be identical. Common factor one last time to get an expression in standard form.

Example: Factor $4x^2 - 8x - 5$

$(4)(-5) = -20$ $\quad (2)(-10) = -20$

$b = -8$ $\quad (2) + (-10) = -8$

$4x^2 - 8x - 5$

$= \underbrace{4x^2 + 2x}\underbrace{-10x - 5}$

$= 2x(2x + 1) - 5(2x + 1)$

$= (2x + 1)(2x - 5)$

order of binomials does not matter.

More Examples:

Factor each of the following.

1) $12x^2 + 17x - 7$

$ac = (12)(-7)$
$= -84$
$b = 17$
$(21)(-4) = -84$
$(21) + (-4) = 17$

$= \underbrace{12x^2 - 4x}\underbrace{+ 21x - 7}$
$= 4x(3x-1) + 7(3x-1)$
$= (4x+7)(3x-1)$

2) $x^2 - 3x + 2$

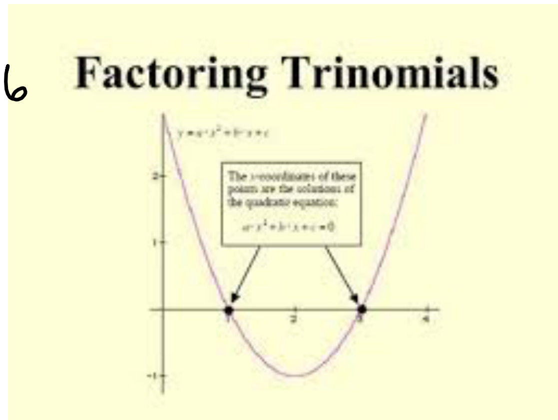
$ac = (1)(2)$
$= 2$
$b = -3$
$(-1)(-2) = 2$
$(-1) + (-2) = -3$

$= \underbrace{x^2 - x}\underbrace{-2x + 2}$
$= x(x-1) - 2(x-1)$
$= (x-2)(x-1)$

3) $4x^2 + 10x + 6$ ← CF!

$ac = (2)(3) = 6$
$b = 5$
$(2)(3) = 6$
$(2) + (3) = 5$

$= 2(2x^2 + 5x + 3)$
$= 2[\underbrace{2x^2 + 3x}\underbrace{+ 2x + 3}]$
$= 2[x(2x+3) + 1(2x+3)]$
$= 2(2x+3)(x+1)$



How is Decomposition Related to the Distributive Property?

The other day ~~On Monday~~ we talked about the idea that the distributive property and common factoring were opposite operations.

$$a(b + c) \underset{\text{Common Factoring}}{\overset{\text{Distributive Property}}{\rightleftarrows}} ab + ac$$

Applying the distributive property twice is the opposite operation to decomposition.

Factor with Decomposition ↑

$(2x - 1)(x + 4)$
$= 2x(x + 4) - 1(x + 4)$
$= 2x^2 + 8x - 1x - 4$
$= 2x^2 + 7x - 4$

↓ Expand & Simplify

This means that you can use the ability to expand and simplify to check your answers when you factor!

Go back and check your answers from the examples above.