Date: ____________________

3.4 Expanding Quadratic Expressions

The **distributive property** is the process of multiplying a monomial with a polynomial. You spent lots of time on this in grade nine when you were discussing polynomials.

Distributive Property: $a(b + c) = ab + ac$

Things to Remember:

- You can only add/subtract "like terms", or terms with the same variable AND exponent!
- **The sign in front of a term is connected to it, and therefore moves with it when you collect like terms.**
- If you have to multiply variables together, multiply the coefficients and add the exponents. ex/ $2x^1(x^1 - 5) = 2x^2 - 10x$

The Distributive Property with a Twist

This year you need to be able to multiply two binomials together. To do this, you simply apply the distributive property twice.

ex/ $(a + b)(c + d)$
$= a(c + d) + b(c + d)$
$= ac + ad + bc + bd$

To do this with actual algebraic expressions, you need to multiply the terms in the second bracket by each term in the first bracket.

ex/ $(2x - 5)(x + 3)$
$= 2x(x + 3) - 5(x + 3)$
$= 2x^2 \boxed{+ 6x - 5x} - 15$
standard form → $= 2x^2 + x - 15$

Collect like terms here to finish the question!

Try these questions on your own. Be sure to show the steps of the distributive property (no shortcuts yet!). Solutions will appear in about 5 minutes.

1) $(x - 5)(x - 5)$

$= x(x-5) - 5(x-5)$

$= x^2 \boxed{-5x - 5x} + 25$

$= x^2 - 10x + 25$

2) $(3x - 5)(2x + 5)$

$= 3x(2x+5) - 5(2x+5)$

$= 6x^2 \boxed{+15x - 10x} - 25$

$= 6x^2 + 5x - 25$

3) $(x - 5)(x + 5)$

$= x(x+5) - 5(x+5)$

$= x^2 \boxed{+5x - 5x} - 25$ → equals zero!

$= x^2 - 25$

This method of multiplying can be applied to any pair of polynomials that you are trying to multiply.

General Rule for Expanding Polynomials:

Multiply everything in the second bracket by each term in the first.

For example, to expand and simplify $(x^2 - 5x + 3)(4x^2 - 6x + 1)$, you would multiply the entire second bracket by each term in the first bracket.

$$(x^2 - 5x + 3)(4x^2 - 6x + 1)$$

$$= x^2(4x^2 - 6x + 1) - 5x(4x^2 - 6x + 1) + 3(4x^2 - 6x + 1)$$

$$= 4x^4 \boxed{-6x^3} \boxed{+x^2} \boxed{-20x^3} \boxed{+30x^2} \boxed{-5x} \boxed{+12x^2} \boxed{-18x} + 3$$

$$= 4x^4 - 26x^3 + 43x^2 - 23x + 3$$

What happens if we have an 'a' value that is not one?

In all of the previous examples, there has not been a number in front of the brackets. When there is a number out there you have two choices:

Example: Expand and simplify $2(x - 4)(3x - 5)$

$2(3)(5)$
$= 30$

Option 1:

- Multiply the number in front **into the first bracket only**.

- Expand and simplify.

$2(x - 4)(3x - 5)$
$= (2x - 8)(3x - 5)$
$= 2x(3x - 5) - 8(3x - 5)$
$= 6x^2 - 10x - 24x + 40$
$= 6x^2 - 34x + 40$

Option 2:

- Expand and simplify the brackets as before. Ignore the number in front for now.
- Multiply the number with all terms at the end.

$2(x - 4)(3x - 5)$
$= 2[x(3x - 5) - 4(3x - 5)]$
$= 2(3x^2 - 5x - 12x + 20)$
$= 2(3x^2 - 17x + 20)$
$= 6x^2 - 34x + 40$

It does not matter if you use option 1 or option 2. The answer is the same, so pick the one that works for you, or the one that best suits the situation!

Practice Questions:

Expand and simplify each of the following. Compare your answer with the solutions that will appear soon.

1) $3(x - 4)(2x + 5)$

$= (3x - 12)(2x + 5)$
$= 3x(2x + 5) - 12(2x + 5)$
$= 6x^2 + 15x - 24x - 60$
$= 6x^2 - 9x - 60$

2) $\frac{1}{2}(4x - 5)(x + 2)$

$= \frac{1}{2}[4x(x + 2) - 5(x + 2)]$
$= \frac{1}{2}(4x^2 + 8x - 5x - 10)$
$= \frac{1}{2}(4x^2 + 3x - 10)$
$= 2x^2 + \frac{3}{2}x - 5$

A Shortcut for Multiplying Binomials

To quickly multiply binomials, you can use the acronym **FOIL**.

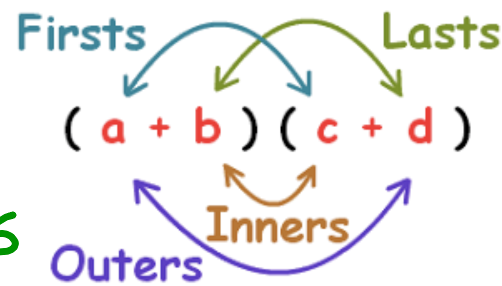
Multiply the terms in this order!

First, Inner, Outer, Last

ex/ $(2x - 5)(x + 1)$

$= 2x^2 + 2x - 5x - 5$

$= 2x^2 - 3x - 5$



You can use this shortcut if you want to. The advantages to expanding with the distributive property (the long way) are:
- it works all of the time, not just for binomials;
- it is the opposite of decomposition (process used to factor in chapter 4) so it can help you to check your work.

Practice Questions:

Use FOIL to expand and simplify each of the following expressions.

1) $(2x - 5)(x + 7)$

$= 2x^2 + 14x - 5x - 35$

$= 2x^2 + 9x - 35$

2) $(3x - 10)(2x + 3)$

$= 6x^2 + 9x - 20x - 30$

$= 6x^2 - 11x - 30$

3) $(x - 5)(x + 5)$

$= x^2 + 5x - 5x - 25$

$= x^2 - 25$

The expanded form of a quadratic relation will have the form

$$y = ax^2 + bx + c$$

This is called **STANDARD FORM**. So now we have seen two of the three forms for writing quadratic relations!