Calculus Appendix: Related Rates

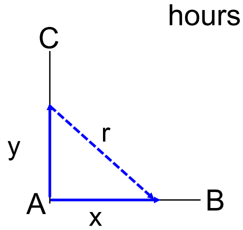
Now that you know what implicit differentiation is, we can apply derivatives to real world problems. Often **two quantities change in relation to each other.**

For example, we solved lots of motion problems where one thing (usually a train) left from a fixed point, and another arrived at it. In real life, this happens sometimes, but we also will encounter two things moving relative to one another (what if two trains leave from the same spot and go in different directions??).

Also, when you are examining the rate of change of volume in a container that is not cylindrical you need to be aware that the rate at which the volume changes is dependent on the radius at that particular height (we did lots with prisms and cylinders where the dimensions are constant, but what if we are filling a cone with water??).

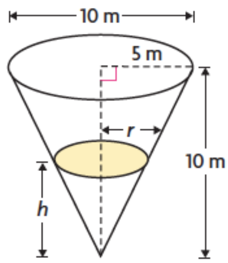
In order to solve these problems, we need to set up equations using the chain rule and use implicit differentiation, as we have two variables that are changing.

Example 1: Two cars start from point A and travel along perpendicular roads AB and AC (as shown below). The first car drives at a speed of 45 km/h along AB and the second travels at a speed of 40 km/h along AC. If the second car starts driving 1 hour before the first, at what rate are their cars separating 3 hours after the first car leaves?



Let x be the distance that the first car has travelled along AB and y be the distance that the second car has travelled along AC.

Example 2: Water is pouring into an inverted right circular cone at a rate of π m^3/min. The height and the diameter of the base of the cone are both 10 m. How fast is the water level rising when the depth of the water is 8 m?



Practice Problems:

PART A

1. Express the following statements in symbols:
   a. The area, A, of a circle is increasing at a rate of 4 m^2/s.
   b. The surface area, S, of a sphere is decreasing at a rate of 3 m^2/min.
   c. After travelling for 15 min, the speed of a car is 70 km/h.
   d. The x- and y-coordinates of a point are changing at equal rates.
   e. The head of a short-distance radar dish is revolving at three revolutions per minute.

PART B

2. The function $T(x) = \frac{200}{1 + x^2}$ represents the temperature, in degrees Celsius, perceived by a person standing x metres from a fire.
   a. If the person moves away from the fire at 2 m/s, how fast is the perceived temperature changing when the person is 5 m away?
   b. Using a graphing calculator, determine the distance from the fire when the perceived temperature is changing the fastest.
   c. What other calculus techniques could be used to check the result?

4. Each edge of a cube is expanding at a rate of 4 cm/s.
   a. How fast is the volume changing when each edge is 5 cm?
   b. At what rate is the surface area changing when each edge is 7 cm?

6. The area of a circle is decreasing at the rate of 5 m^2/s when its radius is 3 m.
   a. At what rate is the radius decreasing at that moment?
   b. At what rate is the diameter decreasing at that moment?

7. Oil that is spilled from a ruptured tanker spreads in a circle. The area of the circle increases at a constant rate of 6 km^2/h. How fast is the radius of the spill increasing when the area is 9π km^2?

8. The top of a 5 m wheeled ladder rests against a vertical wall. If the bottom of the ladder rolls away from the base of the wall at a rate of $\frac{1}{3}$ m/s, how fast is the top of the ladder sliding down the wall when it is 3 m above the base of the wall?

9. How fast must someone let out line if a kite is 30 m high, 40 m away horizontally, and continuing to move away horizontally at a rate of 10 m/min?

10. If the rocket shown below is rising vertically at 268 m/s when it is 1220 m up, how fast is the camera-to-rocket distance changing at that instant?