Wednesday, October 24, 2018

9.2 Systems of Equations & 9.5 The Distance from a Point to Line

1) Systems of Equations

We already know how to solve a system of equations in R^2 (substitution, elimination, graphing). We can also use a matrix and elementary row operations to solve in both R^2 and R^3 as long as we have an appropriate number of equations (same # of equations as unknowns).

Interpreting an Algebraic Solution Geometrically

Case 1: The solution is $0 = a$, where a is any constant. What does this mean?

Case 2: The solution is $x = a$, $y = a$, or $z = a$, where a is any constant. What does this mean?

Case 3: The solution is $0 = 0$. What does this mean, and what do we need to do in when this occurs?

Vocabulary Note: A system of equations is consistent if it has a solution (one POI or an infinite number of POIs), and it is inconsistent if it does not have a solution (parallel lines or planes).

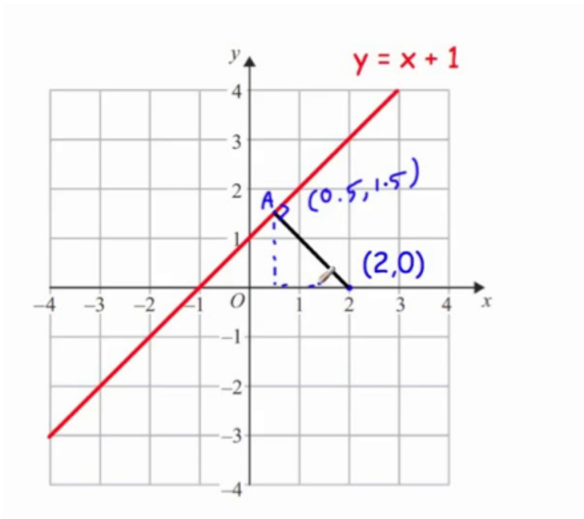
If you really dislike matrices, you can solve using elementary operations (elimination/substitution). We will do one example this way, but I strongly recommend using matrices!!

Example: Solve the system of equations below using elementary operations.

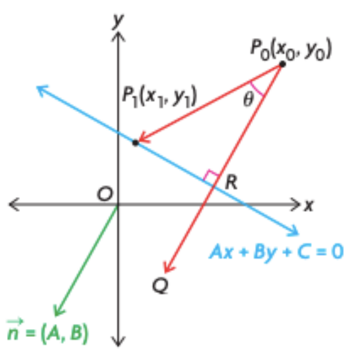
$$x - y + z = 1$$
$$2x + y - z = 11$$
$$3x + y + 2z = 12$$

2) <u>The Distance from a Point to a Line in R^2 and R^3</u>

Remember that the distance from a point to a line (or plane) is always the perpendicular distance. In grade ten, we did this in R^2, but it was a long process. Take a minute to try to figure out how you would find the distance from the point to line shown below.



Fortunately, now that we have some knowledge of vectors, we can use a formula to find the distance from any point to a line in R^2:



$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

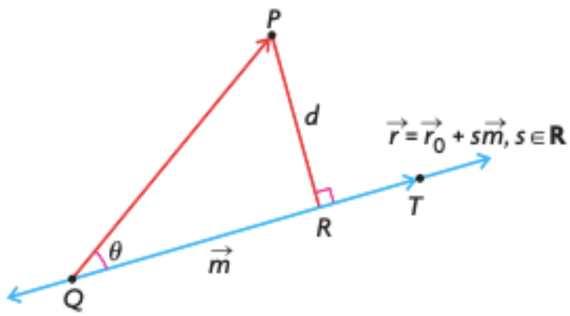
Where (A, B) is the normal vector to the line and (x_o, y_o) is the point given.

A detailed proof of this formula is shown in your text book.

Example: Find the distance from the point P(-3, 5) to the line $4x - y + 3 = 0$.

Why won't this formula work in R^3?

Let's figure out a formula that will work, using what we know and the diagram provided.



Example: Determine the distance from the point P(-1, 1, 6) to the line with equation r = (1, 2, -1) + t(0, 1, 1).