Tuesday, September 11, 2018

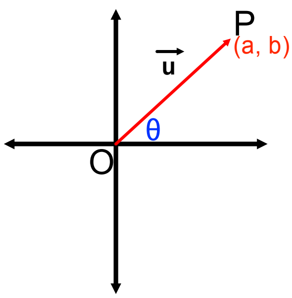
6.5 Vectors in R^2 and R^3

Until now, we have talked about vectors geometrically. The magnitude has either been given, or calculated as a length. The direction has been described in terms of bearings or degrees from the vertical/horizontal.

We can also represent vectors using a Cartesian plane (Cartesian vectors or algebraic vectors). R^2 refers to two-dimensional space (x and y axes).

Algebraic Vectors in R^2

position vector - any vector, $\vec{u}$, with it's tail at the origin
(also represented by (a, b))



The angle, θ, is measured counterclockwise (upward) from the positive x - axis

$\vec{\mathbf{u}} = \overrightarrow{OP} = (a, b)$

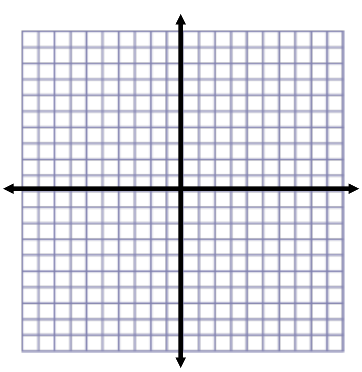
$|\vec{u}| =$

$\theta =$

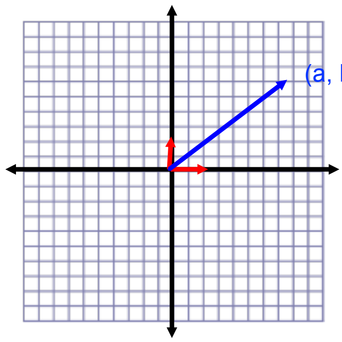
Practice Problems:

1) Given the position vector (2, 5), determine it's magnitude and direction.

2) Given the vector that joins A(-3, 4) and B(5, -1), find its position vector.



Unit Vector Notation: We can also write vector (a, b) in terms of $\vec{i}$ and $\vec{j}$, where $\vec{i}$ and $\vec{j}$ are unit vectors along the x and y axes.

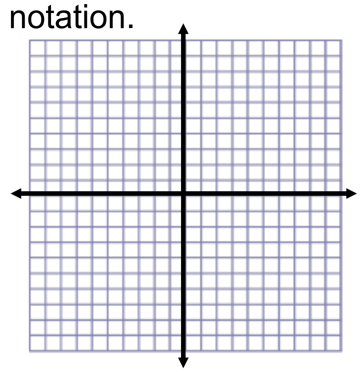


$\vec{\mathbf{u}} = a\vec{i} + b\vec{j}$

$\vec{i} = (1, 0)$

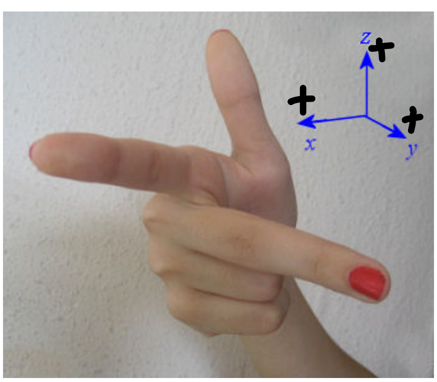
$\vec{j} = (0, 1)$

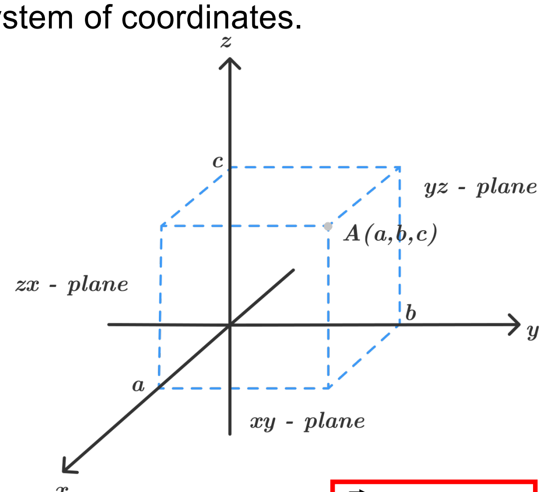
3) Express the position vector P(-4, -2) in unit vector notation.



Algebraic Vectors in R^3

Points (and position vectors) in three dimensions are represented as P(a, b, c). We now have an x, y, and z axis to deal with as well. We locate points in three dimensions using a right - hand system of coordinates.





The unit vector along the z - axis is indicated by k.

$\therefore \vec{\mathbf{u}} = \overrightarrow{OP} = (a, b, c)$ or $\vec{u} = a\vec{i} + b\vec{j} + c\vec{k}$

$\vec{i} = (1, 0, 0)$

$\vec{j} = (0, 1, 0)$

$\vec{k} = (0, 0, 1)$

Plotting Points in Three Dimensions

To plot a **point** in three dimensions:

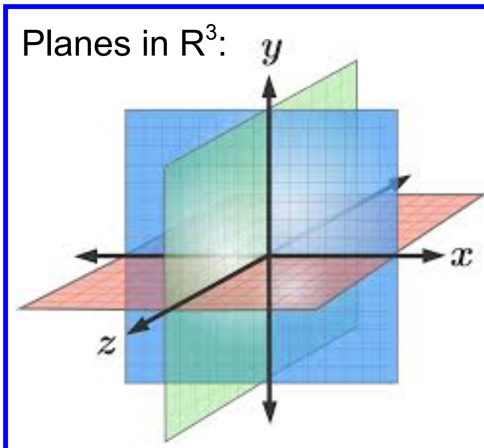
- Draw the component vectors along each axis (plot each piece of the coordinate separately, so if the point is (1, 2, 3), you need to plot (1, 0, 0), (0, 2, 0) and (0, 0, 3)).
- Create a rectangular prism using a ruler. You need to create a rectangle in each plane. Using the example from above, your rectangle in the xy-plane would have vertices at (1, 0, 0), (0, 0, 0), (0, 2, 0) and (1, 2, 0).
- The point you were actually plotting is the one that is not in the same plane as O.

To draw a **position vector** in R^3:

- Plot the point P(a, b, c) using the process described above.
- Create a position vector by attached the origin, O(0, 0, 0), and P(a, b, c).

Unfortunately there is no shortcut to plotting points in three-space. Please practice this skill so that you can do it on an assessment!

Planes in R^3:



4) Draw a set of x, y, and z - axes and plot A(3, 2, -4). Draw $\overrightarrow{OA}$ and write the vector in terms of its components. Use a rectangular prism to show the components on your diagram.