6.8 Linear Combinations and Spanning Sets

1. All 3 vectors have the same direction (collinear), so they form a line, not a plane.
2. These vectors span the xy-plane, but the z-component is always zero, so they cannot span R^3.
3. $m(0,1)$ or $(0,m)$ where $m \in \mathbb{R}$, so $(0,1)$ and $(0,-1)$ span the same set.
4. Set: $m(1,0,0)$ Other vectors: $(2,0,0)$, $(-5,0,0)$, etc.
5. $(0,0)$ cannot be used in a spanning set for R^2
6. $\{(-1,2),(-1,1)\}$ (There are other right answers)

7a) $2(2\vec{a}-3\vec{b}+\vec{c})-4(-\vec{a}+\vec{b}-\vec{c})+(\vec{a}-\vec{c})$
$=4\vec{a}-6\vec{b}+2\vec{c}+4\vec{a}-4\vec{b}+4\vec{c}+\vec{a}-\vec{c}$
$=9\vec{a}-10\vec{b}+5\vec{c}$
$=9(\vec{i}-2\vec{j})-10(\vec{j}-3\vec{k})+5(\vec{i}-3\vec{j}+2\vec{k})$
$=9\vec{i}-18\vec{j}-10\vec{j}+30\vec{k}+5\vec{i}-15\vec{j}+10\vec{k}$
$=14\vec{i}-43\vec{j}+40\vec{k}$

b) $\frac{1}{2}(2\vec{a}-4\vec{b}-8\vec{c})-\frac{1}{3}(3\vec{a}-6\vec{b}+9\vec{c})$
$=\vec{a}-2\vec{b}-4\vec{c}-\vec{a}+2\vec{b}-3\vec{c}$
$=-7\vec{c}$
$=-7(\vec{i}-3\vec{j}+2\vec{k})$
$=-7\vec{i}+21\vec{j}-14\vec{k}$

8. $\{(1,0,0),(0,1,0)\}$ span the xy-plane in R^3

Lots of options, these are just the easiest to work with.

For $(-1,2,0)$:
$m(1,0,0)+n(0,1,0)=(-1,2,0)$
$(m,0,0)+(0,n,0)=(-1,2,0)$
$(m,n,0)=(-1,2,0)$
$m=-1,\ n=2$
$\therefore -(1,0,0)+2(0,1,0)=(-1,2,0)$

For $(3,4,0)$:
$m(1,0,0)+n(0,1,0)=(3,4,0)$
$(m,0,0)+(0,n,0)=(3,4,0)$
$(m,n,0)=(3,4,0)$
$m=3,\ n=4$
$\therefore 3(1,0,0)+4(0,1,0)=(3,4,0)$

9a) This set produces the xy-plane.

b) $m(1,0,0)+n(0,1,0)=(-2,4,0)$
$(m,n,0)=(-2,4,0)$
$-2(1,0,0)+4(0,1,0)=(-2,4,0)$

c) The z-component is zero. We cannot multiply 0 by any value to produce 8. Also, the vector $(3,5,8)$ is not on the xy-plane, so it cannot be represented by a linear combination of $\{(1,0,0),(0,1,0)\}$

d) Still the xy-plane.

10. $2(a,3,c)+3(c,7,c)=(5,b+c,15)$
$(2a+3c,\ 6+21,\ 2c+3c)=(5,b+c,15)$
$\therefore 2a+3c=5,\quad 27=b+c,\quad 5c=15$
$2a+3(5)=5 \quad 27=b+3 \quad c=3$
$2a=-10 \quad 24=b$
$a=-5$
$a=-5,\ b=24,\ c=3$

11. $a(-1,3)+b(1,5)=(-10,-34)$
$(-a+b,\ 3a+5b)=(-10,-34)$

① x-component:
$-a+b=-10$
$b=-10+a \xrightarrow{\text{sub into ②}}$
$b=-10+2$
$b=-8$

② y-component:
$3a+5b=-34$
$3a+5(-10+a)=-34$
$3a-50+5a=-34$
sub into ① $8a=16$
$a=2$

$\therefore 2(-1,3)-8(1,5)=(-10,-34)$

12a) $a(2,-1)+b(-1,1)=(x,y)$
$(2a-b,\ -a+b)=(x,y)$
$2a-b=x$ ①
$+\ -a+b=y$ ②
add ①+② to eliminate b.
$\boxed{a=x+y}$ ← sub in to ② to get an expression for b.
$-(x+y)+b=y$
$b=y+x+y$
$\boxed{b=x+2y}$

13a) See if it is possible to write one as a linear combination of the other two.
$m(-1,2,3)+n(4,1,-2)\overset{?}{=}(-14,-1,16)$
$(-m+4n,\ 2m+n,\ 3m-2n)=(-14,-1,16)$
① x-component: $-m+4n=-14$ ② y-component: $2m+n=-1$ ③ z-component: $3m-2n=16$
Make a linear system with any 2 equations:
$2\times$① $-2m+8n=-28$
Add! + ② $2m+n=-1$
$9n=-29$
Sub $n=-29/9$ into ①
$-m+4\left(\frac{-29}{9}\right)=-14$
$\boxed{m=10/9}$
Check these values of m and n in ③ to see if they work.
$3(10/9)-2(-29/9)\overset{?}{=}16$
$88/9\neq 16$
$\therefore$ (-14, 1, 16) does not lie in the same plane as (-1,2,3) and (4,1,-2)

b) For (2, -3):
$a=2-3$ $b=2+2(-3)$
$=-1$ $=-4$
$\therefore -1(2,-1)-4(-1,1)=(2,-3)$
For (124, -5):
$a=124-5$ $b=124+2(-5)$
$=119$ $=114$
$\therefore 119(2,-1)+114(-1,1)=(124,-5)$
For (4, -11):
$a=4-11$ $b=4+2(-11)$
$=-7$ $=-18$
$\therefore -7(2,-1)-18(-1,1)=(4,-11)$

13b) $m(-1,3,4)+n(0,-1,1)=(-3,14,7)$
$(-m,\ 3m-n,\ 4m+n)=(-3,14,7)$

x-comp	y-comp	z-comp
$-m=-3$	$3m-n=14$	$4m+n\overset{?}{=}7$
$\boxed{m=3}$	$3(3)-n=14$	$4(3)+(-5)\overset{?}{=}7$
	$\boxed{-5=n}$	$7=7$
Solve here		Check here

$\therefore$ The vectors are coplanar.
$3(-1,3,4)-5(0,-1,1)=(-3,14,7)$

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14. $\vec{OA} = (-1, 3, 4)$ $\vec{OB} = (-2, 3, -1)$ $\vec{OC} = (-5, 6, x)$

$m(-1, 3, 4) + n(-2, 3, -1) = (5, 6, x)$

$(-m - 2n, 3m + 3n, 4m - n) = (5, 6, x)$

① x-comp ② y-comp ③ z-comp

$-m - 2n = -5$ $(3m + 3n = 6) \div 3$ $4m - n = x$

add ① $m + n = 2$ ← $m + n = 2$

$-n = -3$

$n = 3$

$-m - 2(3) = -5$

$-m = 1$

$m = -1$

Sub into ③ → $4(-1) - 3 = x$

$-7 = x$

$\boxed{\therefore x = -7.}$

15. If they span R^2, they cannot be parallel. The only way to satisfy this equation is to use coefficients of zero.
$\therefore m = 2, n = -3.$

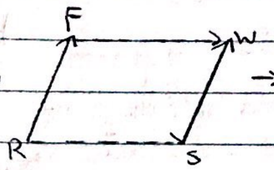
Review Exercise

1a) $|\vec{a}+\vec{b}| \geq |\vec{a}|$ - True in most cases, but not all. This is false if $\vec{b} = -\vec{a}$, because then $|\vec{a}+\vec{b}| = |\vec{a}+(-\vec{a})|$, which is zero. This is less than $|\vec{a}|$.

b) $|\vec{a}+\vec{b}| = |\vec{a}+\vec{c}|$ implies $|\vec{b}| = |\vec{c}|$ - True because magnitude is a scalar quantity. $\vec{a}$ is one side of a parallelogram for $|\vec{a}+\vec{b}|$ to equal $|\vec{a}+\vec{c}|$, $|\vec{b}|$ and $|\vec{c}|$ would have to be the same.

c) $\vec{a}+\vec{b} = \vec{a}+\vec{c}$ implies that $\vec{a} = \vec{c}$ → true. If $\vec{a}+\vec{b} = \vec{a}+\vec{c}$, then $\vec{a}-\vec{a}+\vec{b} = \vec{a}-\vec{a}+\vec{c}$ (subtract $\vec{a}$ from both sides) $\therefore \vec{b} = \vec{c}$

d) $\vec{RF} = \vec{SW}$ implies $\vec{RS} = \vec{FW}$



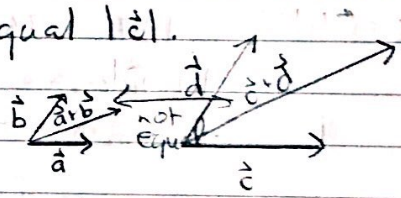
→ True - each pair would represent opposite sides of the parallelogram formed by RFWS.

e) $m\vec{a} + n\vec{a} = (m+n)\vec{a}$ → True - distributive law ("common factored" $\vec{a}$ out)

ex/ $2\vec{a} + 3\vec{a} = (2+3)\vec{a}$

$5\vec{a} = 5\vec{a}$ ☺

f) If $|\vec{a}| = |\vec{b}|$ and $|\vec{c}| = |\vec{d}|$, then $|\vec{a}+\vec{b}| = |\vec{c}+\vec{d}|$ → False. $|\vec{a}|$ does not necessarily equal $|\vec{c}|$.



2a) $2\vec{x} - 3\vec{y} + 5\vec{z}$

Sub in for $\vec{x}, \vec{y}$ and $\vec{z}$

$= 2(2\vec{a}-3\vec{b}-4\vec{c}) - 3(-2\vec{a}+3\vec{b}+3\vec{c}) + 5(2\vec{a}-3\vec{b}+5\vec{c})$

$= 4\vec{a} - 6\vec{b} - 8\vec{c} + 6\vec{a} - 9\vec{b} - 9\vec{c} + 10\vec{a} - 15\vec{b} + 25\vec{c}$

$= 20\vec{a} - 30\vec{b} + 8\vec{c}$

In this case, simplify first, then sub in.

b) $3(-2\vec{x} - 4\vec{y} + \vec{z}) - (2\vec{x} - \vec{y} + \vec{z}) - 2(-4\vec{x} - 5\vec{y} + \vec{z})$

$= -6\vec{x} - 12\vec{y} + 3\vec{z} - 2\vec{x} + \vec{y} - \vec{z} + 8\vec{x} + 10\vec{y} - 2\vec{z}$

$= -\vec{y}$

$= -(-2\vec{a} + 3\vec{b} + 3\vec{c})$

$= 2\vec{a} - 3\vec{b} - 3\vec{c}$

3a) $\vec{XY} = (-4+2, 4-1, 8-2)$

$= (-2, 3, 6)$

(*Be sure to subtract x-coord. from y-coord - order matters!)

$|\vec{XY}| = \sqrt{(-2)^2 + (3)^2 + 6^2}$

$= 7$ units

b) $\hat{XY} = \frac{1}{7}(-2, 3, 6)$

Multiplying a vector by one over its magnitude will produce a magnitude of one.

opposite direction to $\vec{MN}$

4a) $\vec{YX} = (-1-5,\ 2-5,\ 6-12)$
$= (-6, -3, -6)$

b) $|\vec{YX}| = \sqrt{(-6)^2+(-3)^2+(-6)^2}$
$= 9$ units

$\hat{YX} = \frac{1}{9}(-6,-3,-6)$

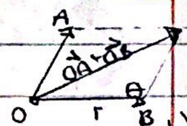
distribute $\frac{1}{9}$ through.
$= \left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$

5. $\vec{NM} = (2-8,\ 3-1,\ 5-2)$
$= (-6, 2, 3)$

$|\vec{NM}| = \sqrt{(-6)^2+2^2+3^2}$
$= 7$ units

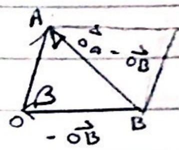
$\hat{NM} = \frac{1}{7}(-6,2,3)$
$= (-6/7,\ 2/7,\ 3/7)$

6a) One diagonal is $\vec{OA}+\vec{OB}$.



$\vec{OA}+\vec{OB} = (3-6,\ 2+6,\ -6-2)$
$= (-3, 8, -8)$

The other is $\vec{OA}-\vec{OB}$.



$\vec{OA}-\vec{OB} = (3+6,\ 2-6,\ -6+2)$
$= (9, -4, -4)$

b) $|\vec{OA}| = \sqrt{3^2+2^2+(-6)^2}$
$= 7$ units

$|\vec{OB}| = \sqrt{6^2+6^2+2^2}$
$= \sqrt{76}$ units

$|\vec{OA}+\vec{OB}| = \sqrt{(-3)^2+(8)^2+(-8)^2}$
$= \sqrt{137}$ units

Use cosine law to find θ:

$|\vec{OA}+\vec{OB}|^2 = |\vec{OA}|^2 + |\vec{OB}|^2 - 2|\vec{OA}||\vec{OB}|\cos\theta$

$137 = 76 + 49 - 2(7)(\sqrt{76})\cos\theta$

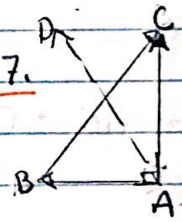
$14\sqrt{76}\cos\theta = -12$

$\cos\theta = -\frac{6}{7\sqrt{76}}$

$\theta = 95.6°$

b) $\beta = 180 - 95.6°$ ← C-pattern, add to 180°.
$= 84.4°$

7.



(Random drawing, not to scale!!)

a) $\vec{AB} = (3, -1, 2)$ $|\vec{AB}| = \sqrt{14}$
$\vec{BC} = (1, 3, -7)$ $|\vec{BC}| = \sqrt{59}$
$\vec{AC} = (4, 2, -5)$ $|\vec{AC}| = \sqrt{45}$

To show it is a right triangle, show that the Pythagorean theorem work.

$|\vec{AB}|^2 + |\vec{AC}|^2 = |\vec{BC}|^2$
$14 + 45 = 59$ ☺

∴ It is a right triangle.

b) $A = \frac{bh}{2}$
$= \frac{|\vec{AB}||\vec{AC}|}{2}$
$= \frac{(\sqrt{14})(\sqrt{45})}{2}$
$= 12.5$ units squared

7c) $P = \sqrt{14} + \sqrt{59} + \sqrt{45}$
$= 18.13$ units

d) $\vec{AB} + \vec{AC} = \vec{AD}$
$(3,-1,2) + (4,2,-5) = \vec{AD}$
$(7, 1, -3) = \vec{AD}$

BUT to get D's coordinates:

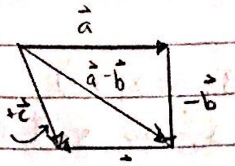
$\vec{OA} = (-1,1,1)$
$\vec{OD} = (x_1, y_1, z_1)$

$\vec{AD} = (x_1+1,\ y_1-1,\ z_1-1)$

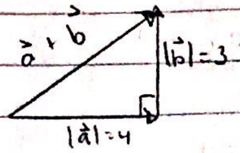
So $x_1+1 = 7$ $y_1-1 = 1$ $z_1-1 = -3$
$x_1 = 6$ $y_1 = 2$ $z_1 = -2$

So $\vec{OD} = (6, 2, -2)$ and $D(6, 2, -2)$

8a) $\vec{a} - \vec{b} + \vec{c}$



b)



Pythagorean → Theorem. $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$

$|\vec{a}+\vec{b}| = \sqrt{16+9}$

$|\vec{a}+\vec{b}| = 5$ units

9. For $\vec{p} = (-11, 7)$

$a(-3,1) + b(-1,2) = (-11,7)$

$(-3a - b,\ a + 2b) = (-11,7)$

① x-comp: $-3a - b = -11$ ② y-comp: $a + 2b = 7$

$a = \boxed{-2b + 7}$

$-3(-2b+7) - b = -11$

$6b - 21 - b = -11$

$5b = 10$

$b = 2$

$a = -2(2) + 7$

$a = 3$

$\therefore 3\vec{q} + 2\vec{r} = \vec{p}$

For $\vec{q} = (-3,1)$

$a(-11,7) + b(-1,2) = (-3,1)$

$(-11a - b,\ 7a + 2b) = (-3,1)$

① x-comp: $-11a - b = -3$ ② y-comp: $7a + 2b = 1$

$b = \boxed{-11a + 3}$

$7a + 2(-11a+3) = 1$

$7a - 22a + 6 = 1$

$-15a = -5$

$a = \frac{1}{3}$

$b = -11(\frac{1}{3}) + 3$

$= -\frac{11}{3} + 3$

$= -\frac{2}{3}$

$\therefore \frac{1}{3}\vec{p} - \frac{2}{3}\vec{r} = \vec{q}$

For $\vec{r} = (-1,2)$

$a(-11,7) + b(-3,1) = (-1,2)$

$(-11a - 3b,\ 7a + b) = (-1,2)$

① x-comp: $-11a - 3b = -1$ ② y-comp: $7a + b = 2$

$b = \boxed{2 - 7a}$

$-11a - 3(2 - 7a) = -1$

$-11a - 6 + 21a = -1$

$10a = 5$

$a = \frac{1}{2}$

$b = 2 - 7(\frac{1}{2})$

$= -3/2$

$\therefore \frac{1}{2}\vec{p} - \frac{3}{2}\vec{q} = \vec{r}$

10a) $\vec{AB} = (1-2,\ 2+1,\ -3-3)$

$= (-1, 3, -6)$

$M_{AB} = \left(\frac{1+2}{2}, \frac{2-1}{2}, \frac{-3+3}{2}\right)$

$= (\frac{3}{2}, \frac{1}{2}, 0)$

#10 is more of a Ch. 8/9 question (don't stress!)

$ax + by + cz + d = 0$

$-1(\frac{3}{2}) + 3(\frac{1}{2}) - 6(0) + d = 0$

$d = 0$

$-x + 3y - 6z = 0$

$\cdot (x, y, z)$ is any point on the plane.

b) Sub in values for 2 variables & solve for the third ☺

11a) $2(a, b, 4) + \frac{1}{2}(6, 8, c) - 3(7, c, -4) = (-24, 3, 25)$

$(2a, 2b, 8) + (3, 4, \frac{1}{2}c) + (-21, -3c, 12) = (-24, 3, 25)$

$(2a - 18,\ 2b - 3c + 4,\ 20 + \frac{1}{2}c) = (-24, 3, 25)$

$\therefore 2a - 18 = -24$

$2a = -6$

$a = -3$

$20 + \frac{1}{2}c = 25$

$\frac{1}{2}c = 5$

$c = 10$

$2b - 3c + 4 = 3$

$2b - 30 = -1$

$2b = 29$

$b = 29/2$

b) $2(a, a, \frac{1}{2}a) + (3b, 0, -5c) + 2(c, \frac{3}{2}c, 0) = (3, -22, 54)$

$(2a, 2a, a) + (3b, 0, -5c) + (2c, 3c, 0) = (3, -22, 54)$

$(2a + 3b + 2c, \underbrace{2a + 3c, a - 5c}) = (3, \underbrace{-22, 54})$

start here!

② $2a + 3c = -22$ ③ $a - 5c = 54$

$2 \times$ ③ $2a - 10c = 108$ $a - 5(-10) = 54$

subtract ⊖ $13c = -130$ $a + 50 = 54$

$\boxed{c = -10 \quad a = 4}$ Put these in to ① to solve for b.

① $2(4) + 3b + 2(-10) = 3$

$8 + 3b - 20 = 3$

$3b = 15$

$b = 5$

$\therefore a = 4, b = 5, c = -10.$

12a) $\vec{AB} = (2-1, 2+1, 2-1)$ $|\vec{AB}| = \sqrt{1^2 + 3^2 + 1^2}$

$= (1, 3, 1)$ $= \sqrt{11}$

$\vec{BC} = (4-2, -2-2, 1-2)$ $|\vec{BC}| = \sqrt{2^2 + 4^2 + 1^2}$

$= (2, -4, -1)$ $= \sqrt{21}$

$\vec{AC} = (4-1, -2+1, 1-1)$ $|\vec{AC}| = \sqrt{3^2 + 1^2 + 0^2}$

$= (3, -1, 0)$ $= \sqrt{10}$

$|\vec{AB}|^2 + |\vec{AC}|^2 = |\vec{BC}|^2$

$\therefore$ It is a right triangle

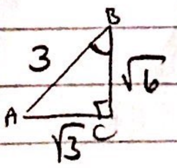
b) The points P, Q and R are collinear because they are scalar multiples of one another (same direction, different magnitudes.)

13a) $\vec{AB} = (1-3, 2-0, 5-4)$ $|\vec{AB}| = \sqrt{2^2 + 2^2 + 1^2}$

$= (-2, 2, 1)$ $= 3$

$\vec{BC} = (2-1, 1-2, 3-5)$ $|\vec{BC}| = \sqrt{1^2 + 1^2 + 2^2}$

$= (1, -1, -2)$ $= \sqrt{6}$



$\vec{AC} = (2-3, 1-0, 3-4)$ $|\vec{AC}| = \sqrt{1^2 + 1^2 + 1^2}$

$= (-1, 1, -1)$ $= \sqrt{3}$

$\therefore \triangle ABC$ is a right triangle because

$|\vec{BC}|^2 + |\vec{AC}|^2 = |\vec{AB}|^2$

b) Use SOHCAHTOA.

$\cos B = \frac{\sqrt{6}}{3}$

14 a) $\vec{DA}$ and $\vec{BC}$, $\vec{AB}$ and $\vec{CD}$ (opposites because they have equal magnitude and opposite direction - lots of other correct answers!)

b) $\vec{AD}$ and $\vec{BC}$, $\vec{AB}$ and $\vec{DC}$

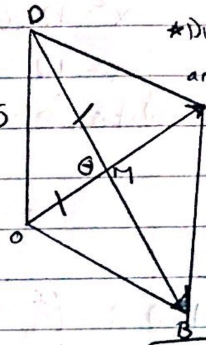
c) $|\vec{AD}|^2 + |\vec{DC}|^2 = |\vec{DB}|^2$ is true because $\vec{AC} = \vec{DB}$ (diagonals of a rectangle are equal in length) and $\vec{AD}$, $\vec{DC}$ and $\vec{AC}$ make a right triangle, so the Pythagorean Theorem holds true.

15 a) C (3, 0, 5) ← out + up, no movement along y.
P (3, 4, 5) ← move in all directions
E (0, 4, 5) ← right + up, no movement along x.
F (0, 4, 0) ← only move along y-axis

b) $\vec{DB} = (3-0, 4-0, 0-5)$
$= (3, 4, -5)$

$\vec{CF} = (0-3, 4-0, 0-5)$
$= (-3, 4, -5)$

c)



*Diagonals of any parallelogram bisect each other

$|\vec{OD}| = 5$

$|\vec{OP}| = \sqrt{3^2+4^2+5^2}$
$= 5\sqrt{2}$

$|\vec{DB}| = \sqrt{3^2+4^2+5^2}$
$= 5\sqrt{2}$

$|\vec{OM}| = \frac{5\sqrt{2}}{2}$ $\therefore |\vec{DM}| = \frac{5\sqrt{2}}{2}$

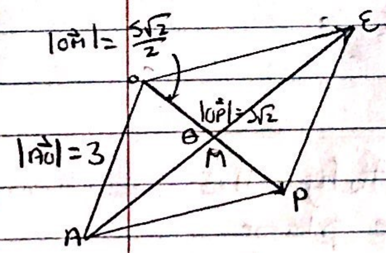
$5^2 = \left(\frac{5\sqrt{2}}{2}\right)^2 + \left(\frac{5\sqrt{2}}{2}\right)^2 - 2\left(\frac{5\sqrt{2}}{2}\right)\left(\frac{5\sqrt{2}}{2}\right)\cos\theta$

$25 = 25 - 25\cos\theta$

$1 = \cos\theta$

$\theta = 90°$. $\therefore$ The diagonals meet at $90°$.

d)



$|\vec{AE}| = \sqrt{(0-3, 4-0, 5-0)}$
$= \sqrt{3^2+4^2+5^2}$
$= 5\sqrt{2}$

$|\vec{AM}| = \frac{5\sqrt{2}}{2}$

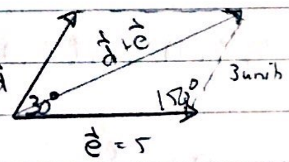
$3^2 = \left(\frac{5\sqrt{2}}{2}\right)^2 + \left(\frac{5\sqrt{2}}{2}\right)^2 - 2\left(\frac{5\sqrt{2}}{2}\right)\left(\frac{5\sqrt{2}}{2}\right)\cos\theta$

$9 = 25 - 25\cos\theta$

$\cos\theta = 16/25$

$\theta = 50.2°$

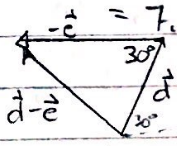
16.



(30° between tails!)

a) $|\vec{d}+\vec{e}|^2 = 3^2+5^2-2(3)(5)\cos 150°$
$= 7.74$ units

b)

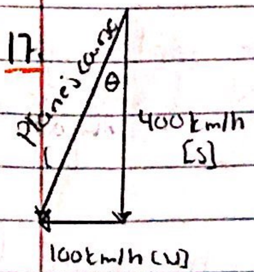


$|\vec{d}-\vec{e}|^2 = 3^2+5^2-2(3)(5)\cos 30°$
$|\vec{d}-\vec{e}| = 2.83$ units

c) $|\vec{e}-\vec{d}| = |\vec{d}-\vec{e}|$ (opposite direction)
$= 2.83$ units.
... change ... the direction.

Scanned with CamScanner

17.



a) Let x be distance travelled. (PT)

$x^2 = 1200^2 + 300^2$

$x = 1236.9$ km

b) $\tan\theta = \frac{1}{4}$

$\theta = 14.04°$ west of south.

18a) {(2,3), (3,5)} is a set of two non-collinear vectors. This means that any vector in R^2 can be expressed as a linear combination of them, so they span R^2.

b) $m(2,3) + n(3,5) = (323, 795)$

$(2m + 3n, 3m + 5n) = (323, 795)$ ① ②

$3\times$① $6m + 9n = 969$

$2\times$② ⊖ $6m + 10n = 1590$

Subtract

$-n = -621$

$\boxed{n = 621}$

$2m + 3(621) = 323$

$2m = -1540$

$\boxed{m = -770}$

19a) $\vec{a} = (5, 9, 14)$, $\vec{b} = (-2, 3, 1)$, $\vec{c} = (3, 1, 4)$

$m(-2, 3, 1) + n(3, 1, 4) = (5, 9, 14)$

$(-2m + 3n, 3m + n, m + 4n) = (5, 9, 14)$

Solve using 2 equations, check in the third.

① x-comp $-2m + 3n = 5$

② y-comp $3m + n = 9$

③ z-comp $m + 4n = 14$

$3\times$③ ⊖ $3m + 12n = 42$

$-11n = -33$

$n = 3$ sub in.

$m + 4(3) = 14$

$m = 2$

Check in ①:

$-2(2) + 3(3) \stackrel{?}{=} 5$

$-4 + 9 = 5$ ☺

} because the values of m + n satisfy all 3 equations, $\vec{a}$, $\vec{b}$, and $\vec{c}$ lie in the same plane.

b) $m(-2, 3, 1) + n(3, 1, 4) = (-13, 36, 23)$

① $-2m + 3n = -13$

② $3m + n = 36$

③ $m + 4n = 23$

$3\times$③ ⊖ $3m + 12n = 69$

$-11n = -33$

$\boxed{n = 3}$

$m + 4(3) = 23$

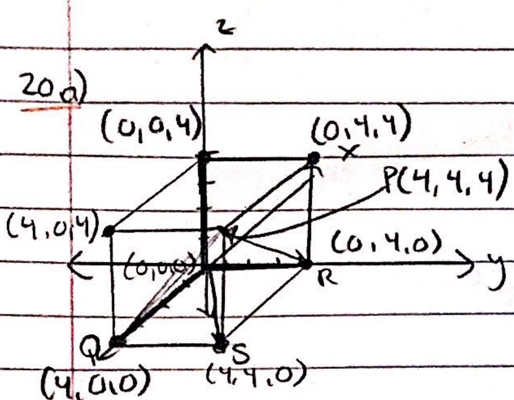
$\boxed{m = 11}$

$-2(11) + 3(3) \stackrel{?}{=} -13$

$-22 + 9 = -13$ ☺

∴ $\vec{a}$ is in the span of $\vec{b}$ + $\vec{c}$ (see part a) for an explanation)

20 a)



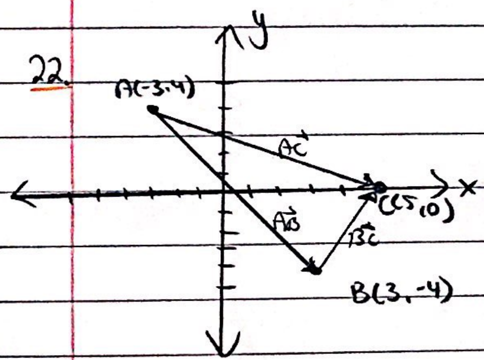
b) $\vec{OP} = (4,4,4)$

c) $\vec{PR} = (0-4, 4-4, 0-4)$
$= (-4, 0, -4)$ ("right" side → see diagram)

d) $\vec{OS} = (4-0, 4-0, 0-0)$
$= (4, 4, 0)$

21. $|2(\vec a+\vec b-\vec c) - (\vec a+2\vec b)+3(\vec a-\vec b+\vec c)|$ ← expand/simplify here first!
$= |2\vec a+2\vec b-2\vec c-\vec a-2\vec b+3\vec a-3\vec b+3\vec c|$
$= |4\vec a-3\vec b+\vec c|$ ← now sub in the vectors given!
$= |4(\vec i+\vec j-\vec k)-3(2\vec i-\vec j+3\vec k)+2\vec i+13\vec k|$
$= |4\vec i+4\vec j-4\vec k-6\vec i+3\vec j-9\vec k+2\vec i+13\vec k|$
$= |7\vec j|$
$= 7$ units

22.



a) $\vec{AB} = (3+3, -4-4)$
$= (6, -8)$
$|\vec{AB}| = \sqrt{6^2+8^2}$
$= 10$ units

$\vec{AC} = (5+3, 0-4)$
$= (8, -4)$
$|\vec{AC}| = \sqrt{8^2+4^2}$
$= \sqrt{80}$ or $4\sqrt{5}$

$\vec{BC} = (5-3, 0+4)$
$= (2, 4)$
$|\vec{BC}| = \sqrt{2^2+4^2}$
$= \sqrt{20}$ or $2\sqrt{5}$

b) If it's a right triangle,
$|\vec{AC}|^2 + |\vec{BC}|^2 = |\vec{AB}|^2$
$20 + 80 = 100$ ✓
∴ It is a right triangle.

23 a) $\vec{FL} = \underbrace{\vec b+\vec a}_{\text{diagonal of the base from F to H}}+\vec c$

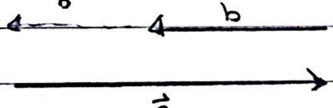
b) $\vec{MK} = \vec a - \vec b$

c) $\vec{HJ} = \underbrace{-\vec a-\vec b}_{\text{diagonal of the base from H to F}}+\vec c$

d) $\vec{IH}+\vec{KJ} = \vec a + (-\vec a)$
$= \vec 0$

e) $\vec{IK} - \vec{IH} = \underbrace{-\vec b+\vec a+\vec c}_{\vec{IK}} - (\vec a)$
$= -\vec b+\vec c$

24.



← if $|\vec a|$ is $2|\vec b|$, then adding $\vec b$ to $\vec a$ will result in $|\vec a+\vec b| = |$

25 a) $|\vec a+\vec b| = \sqrt{|\vec a|^2+|\vec b|^2}$ } ⊥, so right triangle!

b) $|\vec a-\vec b| = \sqrt{|\vec a|^2+|\vec b|^2}$

c) $|2\vec a+3\vec b| = \sqrt{|(2\vec a)|^2+|(3\vec b)|^2}$ ← still Pyth Theorem.
$= \sqrt{4|\vec a|^2+9|\vec b|^2}$

26. If $\vec a$ is ⊥ to $\vec b+\vec c$, $\vec b+\vec c$ form a plane + $\vec a$ is normal to it. The length of $\vec b+\vec c$ won't change that, or the direction.