5.1 Derivatives of Exponential Functions, $y = e^x$

1. The variable is in the exponent, not the base.

2a) $y = e^{3x}$
$y' = 3e^{3x}$

b) $s = e^{3t-5}$
$s' = 3e^{3t-5}$

c) $y = 2e^{10t}$
$y' = 20e^{10t}$

d) $y = e^{-3x}$
$y' = -3e^{-3x}$

e) $y = e^{5-6x+x^2}$
$y' = (2x-6)e^{5-6x+x^2}$

f) $y = e^{\sqrt{x}}$
$y' = \frac{1}{2\sqrt{x}}e^{\sqrt{x}}$

3a) $y = 2e^{x^3}$
$y' = 6x^2e^{x^3}$

e) $h(t) = e^{t^2} + 3e^{-t}$
$h'(t) = 2te^{t^2} - 3e^{-t}$

b) $y = xe^{3x}$
$y' = 3xe^{3x} + e^{3x}$
$= (3x+1)e^{3x}$

f) $g(t) = \dfrac{e^{2t}}{1+e^{2t}}$

$g'(t) = \dfrac{2e^{2t}(1+e^{2t}) - e^{2t}(2e^{2t})}{(1+e^{2t})^2}$ common factor

$= \dfrac{2e^{2t}(1+e^{2t}-e^{2t})}{(1+e^{2t})^2}$

$= \dfrac{2e^{2t}}{(1+e^{2t})^2}$

c) $f(x) = \dfrac{e^{-x^3}}{x}$

$f'(x) = \dfrac{(-3x^2e^{-x^3})(x) - (e^{-x^3})(1)}{x^2}$

$= \dfrac{(-3x^3-1)(e^{-x^3})}{x^2}$

$= -\dfrac{(3x^3+1)}{x^2e^{x^3}}$

6. $y = e^{-x}$, $x = -1$, $y = e$ $(-1, e)$ Pt. of tang.
$y' = -e^{-x}$ ← slope, when $x = -1$, $y' = -e$
$y = mx + b$
$e = -e(-1) + b$
$0 = b$
$\boxed{y = -ex}$

d) $f(x) = \sqrt{x}\, e^x$
$f'(x) = \dfrac{1}{2\sqrt{x}} \cdot e^x + \sqrt{x}e^x$

$= \dfrac{e^x + 2xe^x}{2\sqrt{x}}$

$= \dfrac{e^x(1+2x)}{2\sqrt{x}}$

8. $y = x^2e^{-x}$ when $x = 2$, $y = 4e^{-2}$
$y' = 2xe^{-x} - x^2e^{-x}$ $x = 0$, $y = 0$
$xe^{-x}(2-x) = 0$
$x = 0$, $x = 2$ $(0,0)$ and $(2, \frac{4}{e^2})$

9. $y = \frac{5}{2}(e^{\frac{x}{5}} + e^{-\frac{x}{5}})$

$\quad = \frac{5}{2}e^{x/5} + \frac{5}{2}e^{-\frac{x}{5}}$

$y' = \frac{1}{2}e^{x/5} - \frac{1}{2}e^{-x/5}$

$y'' = \frac{1}{10}e^{x/5} + \frac{1}{10}e^{-x/5}$

Write in terms of y:

$y'' = \frac{1}{10}(e^{x/5} + e^{-x/5})$ ← almost y, but it is $\frac{2}{5}$ of y

$= \frac{1}{10}\left(\frac{2}{5}y\right)$

$= \frac{2}{50}y$

$= \frac{y}{25}$

11(a) $y = -3e^x$

$y' = -3e^x$

$y'' = -3e^x$

b) $y = xe^{2x}$

$y' = 2xe^{2x} + e^{2x}$

$\quad = e^{2x}(2x+1)$

$y'' = 4xe^{2x} + 2e^{2x} + 2e^{2x}$

$\quad = 4xe^{2x} + 4e^{2x}$

$\quad = 4e^{2x}(x+1)$

c) $y = e^x(4-x)$

$\quad = 4e^x - xe^x$

$y' = 4e^x - (xe^x + e^x)$

$\quad = 3e^x - xe^x$

$y'' = 3e^x - (xe^x + e^x)$

$\quad = 2e^x - xe^x$

5.2 The Derivative of the General Exponential Function

1a) $y = 2^{3x}$
$y' = 3(2^{3x})\ln 2$

b) $y = 3.1^x + x^3$
$y' = 3.1^x \ln 3.1 + 3x^2$

c) $s = 10^{3t-5}$
$s' = 3(10^{3t-5})\ln 10$

d) $w = 10^{(5-6n+n^2)}$
$w' = (2n-6)10^{(5-6n+n^2)}\ln 10$

e) $y = 3^{x^2+2}$
$y' = 2x(3^{x^2+2})\ln 3$

f) $y = 400(2)^{x+3}$
$y' = 400(2)^{x+3}\ln 2$

2a) $y = x^5(5^{-x})$
$y' = 5x^4(5^{-x}) + (5^{-x}\ln 5)(x^5)$
$= x^4(5^{-x})(-5 + x\ln 5)$

b) $y = x(3)^{x^2}$
$y' = 3^{x^2} + x(2x)(3^{x^2}\ln 3)$
$= 3^{x^2}(1 + 2x^2\ln 3)$

c) $v = \dfrac{2^t}{t}$
$= \dfrac{(2^t\ln 2)(t) - 2^t}{t^2}$
$= \dfrac{2^t(t\ln 2 - 1)}{t^2}$

2d) $f(x) = \dfrac{\sqrt{3^x}}{x^2}$

$u = 3^x$

$\dfrac{d\sqrt{u}}{du} = \dfrac{1}{2\sqrt{u}}$

$\dfrac{du}{dx} = 3^x\ln 3$

$f'(x) = \dfrac{x^2\left(\frac{1}{2\sqrt{3^x}} \cdot 3^x\ln 3\right) - 2x(\sqrt{3^x})}{x^4}$

$= \dfrac{\frac{3^x\ln 3}{2\sqrt{3^x}} - 2\sqrt{3^x}}{x^3}$

$= \dfrac{3^x x\ln 3 - 4(3^x)}{2x^3\sqrt{3^x}}$

$= \dfrac{3^x(x\ln 3 - 4)}{2x^3\sqrt{3^x}}$

$= \dfrac{3^{x/2}(x\ln 3 - 4)}{2x^3}$

$\dfrac{3^x}{3^{x/2}} = 3^{x/2}$

4. $y = 3(2^x)$ at $x = 3$, $y = 24$
$y' = 3(2^x\ln 2)$ (3, 24) point

When $x = 3$, $y' = 24\ln 2$ ✓ slope

$y = mx + b$
$24 = (24\ln 2)(3) + b$

$24 - 72\ln 2 = b$

$y = mx + b$
$y = (24\ln 2)x + 24 - 72\ln 2$

6. $P(t) = 100(1.2)^{-t}$

a) $50 = 100(1.2)^{-t}$
$0.5 = 1.2^{-t}$
$\ln 0.5 = -t\ln 1.2$
$-\dfrac{\ln 0.5}{\ln 1.2} = t$
$3.80 = t$

b) $P(3.8) = 50$
$(3.8, 50)$
$P'(t) = -100(1.2)^{-t}\ln 1.2$
$P'(3.8) = -9.12\%/y$