

Calculus Appendix - Related Rates

b)  $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$ ,  $\frac{dA}{dt} = 4 \text{ m}^2/\text{s}$

b)  $\frac{dS}{dt} = -3 \text{ m}^2/\text{min}$

c)  $\frac{ds}{dt} = 70 \text{ km/h}$ , when  $t = 0.25 \text{ h}$

d)  $\frac{dy}{dt} = \frac{dx}{dt}$

e)  $\frac{d\theta}{dt} = \frac{6\pi \text{ rad/min}}{60}$   
 $= \frac{\pi}{10} \text{ rad/min}$

2.  $T(x) = \frac{200}{1+x^2}$

a)  $\frac{dT}{dt} = \frac{dT}{dx} \cdot \frac{dx}{dt}$

$= \frac{-400x}{(1+x^2)^2} \cdot (2)$

Let  $x=5$ :  $\frac{dT}{dt} = \frac{-400(5)}{(26)^2} \cdot 2$

$= \frac{-4000}{676}$

$= -5.9^\circ\text{C/s}$

3.  $\frac{dA}{dt} = \frac{dA}{dl} \cdot \frac{dl}{dt}$ ,  $A = l^2$   
 $\frac{dA}{dt} = 2l \cdot \frac{dl}{dt}$ ,  $\frac{dA}{dl} = 2l$

Let  $l=10$ :  $\frac{dA}{dt} = 2(10) \cdot 5$   
 $= 100 \text{ cm}^2/\text{s}$

4.  $V = x^3$

a)  $\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$

$= 3x^2 \cdot (4)$   
 $= 12x^2$

Let  $x=5$ :  $\frac{dV}{dt} = 12(25)$   
 $= 300 \text{ cm}^3/\text{s}$

b)  $SA = 6x^2$

$\frac{dS}{dt} = \frac{dS}{dx} \cdot \frac{dx}{dt}$   
 $= 12x \cdot (4)$   
 $= 48x$

Let  $x=7$ :  $\frac{dS}{dt} = 48(7)$   
 $= 336 \text{ cm}^2/\text{s}$

$\frac{dP}{dt} = \frac{dP}{dl} \cdot \frac{dl}{dt}$ ,  $P = 4l$   
 $= 4 \cdot (5)$   
 $= 20 \text{ cm/s}$

5.  $\frac{dA}{dt} = \frac{dA}{dl} \cdot \frac{dl}{dt}$   
 $= \left( \frac{d}{dt} (l \cdot w + w) \right) \cdot \frac{dl}{dt}$   
 $= \left( \frac{2}{3}l + w \right) (-3)$

$= -2l - 3w$   
 $= -2(50) - 3(20)$   
 $= -40 \text{ cm}^2/\text{s}$

6.  $A = \pi r^2$

a)  $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$   
 $= 2\pi r \cdot \frac{dr}{dt}$

$S = \frac{dA}{dr} \cdot \frac{dr}{dt}$   
 $\frac{10}{\pi(6)} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

b)  $A = \frac{1}{4}\pi d^2$

$\frac{dA}{dt} = \frac{dA}{dd} \cdot \frac{dd}{dt}$   
 $S = \frac{1}{2}\pi d \cdot \frac{dd}{dt}$

$\frac{10}{3\pi} = \frac{dA}{dd} \cdot \frac{dd}{dt}$

$$7. A = \pi r^2$$

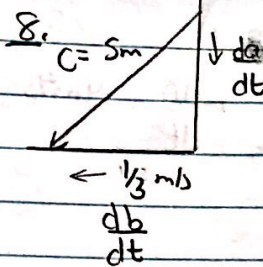
$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

$$= 2\pi r \cdot \frac{dr}{dt}$$

$$9\pi = \pi r^2$$

$$9 = r^2$$

$$3 = r$$



$$c^2 = a^2 + b^2$$

$$a^2 + b^2 = 25$$

$$\frac{d}{dt}(a^2 + b^2) = \frac{d}{dt}(25)$$

$$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} = 0$$

$$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} = 0$$

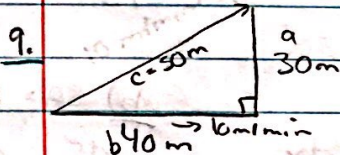
$$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} = 0$$

$$2(3) \cdot \frac{da}{dt} + 2(4) \cdot \frac{db}{dt} = 0$$

$$6 \cdot \frac{da}{dt} + 8 \cdot \frac{db}{dt} = 0$$

$$\frac{da}{dt} = \frac{67}{3}$$

$$= \frac{67}{18} \text{ m/s}$$



$$c^2 = a^2 + b^2$$

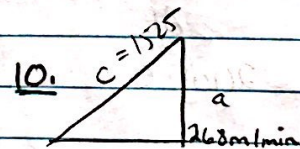
$$\frac{dc^2}{dt} = \frac{d}{dt}(a^2 + b^2) = 2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt}$$

$$2c \cdot \frac{dc}{dt} = 2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt}$$

$$100 \frac{dc}{dt} = 60 \frac{da}{dt} + 80(10) \quad \frac{da}{dt} = 0$$

$$\frac{dc}{dt} = \frac{800}{100}$$

$$= 8 \text{ m/min}$$



b ← set distance (915m)

$$c^2 = a^2 + b^2$$

$$\frac{d}{dt}(c^2) = \frac{d}{dt}(a^2 + b^2) = 2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt}$$

$$2c \cdot \frac{dc}{dt} = 2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt}$$

$$3050 \frac{dc}{dt} = 2440 \cdot \frac{da}{dt} + 1830 \cdot \frac{db}{dt}$$

$$\frac{dc}{dt} = \frac{2440(263)}{3050}$$

$$= 214.4 \text{ m/min}$$