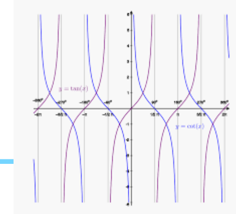
Friday, March 6, 2020

5.5 The Derivative of y = tan x



Bellwork:

Use the quotient identity for tan x to find the derivative of y = tan x.

$$y = \frac{\sin x}{\cos x}$$

$$y' = \frac{(\cos x)(\cos x) - (-\sin x)(\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$y' = \sec^2 x$$

Do you ever HAVE to use the derivative of tan x? Why or why not?

No. We can always rewrite tan x in terms of sin x and cos x.

When will the derivative of tan x be undefined? What type of discontinuity occurs here? When $\cos^2 x = 0$, or when $\cos x = 0$. Vertical asymptote.

Differentiate each of the following functions:

1) $f(x) = \tan^2(2x)$

$= (\tan(2x))^2$

$f'(x) = 2(\tan 2x)(\sec^2 2x)(2)$

$= 4\tan 2x \sec^2 2x$

2) $g(x) = -3x^{-2} + \sin x - 4\tan 2x$

$g'(x) = 6x^{-3} + \cos x - 4(\sec^2 2x)(2)$

$= \dfrac{6}{x^3} + \cos x - 8\sec^2 2x$

3) $h(x) = \underbrace{(x-5)^2}_{1st}\overbrace{(\cos^2 x + 2\tan x)}^{2nd}$

$(\cos x)^2$

$: -2\cos x \sin x$

$h'(x) = 2(x-5)(\cos^2 x + 2\tan x) + (x-5)^2(-2\cos x \sin x + 2\sec^2 x)$

$= 2(x-5)(\cos^2 x + 2\tan x) - 2(x-5)^2(\cos x \sin x - \sec^2 x)$

$= 2(x-5)\left(\cos^2 x + 2\tan x - (x-5)(\cos x \sin x - \sec^2 x)\right)$

4) $y = \sin 2x \tan 2x \cos^2 2x$

$y = \sin 2x \dfrac{\sin 2x}{\cos 2x} \cos^2 2x$ 2nd

$y = \underbrace{\sin^2 2x}_{1st}\overbrace{\cos 2x}$

$y' = 2(\sin 2x)(\cos 2x)(2)(\cos 2x) + \sin^2 2x(-\sin 2x)(2)$

$= 4(\sin 2x)(\cos^2 2x) - 2\sin^3 2x$

$= 2\sin 2x(2\cos^2 2x - \sin^2 2x)$

$2\sin x \cos x = \sin 2x$

$\sin x \cos x = \frac{1}{2}\sin 2x$

Remember that you can use trig identities to write expressions in simpler forms sometimes! Please do that, even if the text book does not. For example, how can you simplify f(x) = tanxcosx? g(x) = cotxsecx?

$f(x) = \tan x \cos x$

$= \dfrac{\sin x}{\cancel{\cos x}} \cancel{\cos x}$

$= \sin x$

$f'(x) = \cos x$

$g(x) = \cot x \sec x$

$= \dfrac{\cancel{\cos x}}{\sin x} \cdot \dfrac{1}{\cancel{\cos x}}$

$= \dfrac{1}{\sin x}$

$g'(x) = \dfrac{-\cos x}{\sin^2 x}$

$= \dfrac{-\cos x}{\sin x} \cdot \dfrac{1}{\sin x}$

$= -\cot x \csc x$

turning point

5) Find the local minimum point on the curve y = 2x - tan x, where x is between negative $\pi/2$ and $\pi/2$.

tangent slope is 0 at a turning point

$y' = 2 - \sec^2 x$

$0 = 2 - \sec^2 x$

$\sec^2 x = 2$

$\dfrac{1}{\cos^2 x} = 2$

$1 = 2\cos^2 x$

$\dfrac{1}{2} = \cos^2 x$

$\pm\dfrac{1}{\sqrt{2}} = \cos x$ ← related acute angle is $\frac{\pi}{4}$

$x = \pm\frac{\pi}{4}$