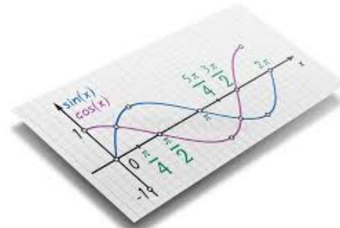
Wednesday, March 4, 2020

5.4 The Derivatives of y = sin x and y = cos x



Bellwork:

In pairs or on your own, use Desmos to:

1) Graph y = sin x. Then open the 'functions' menu and go to miscellaneous. Select "d/dx" and then insert sin x. This will graph the derivative function for sin x. What is the derivative of y = sin x?

$y = \sin x, \ y' = \cos x$

2) Graph y = cos x and repeat the process. What is the derivative of y = cos x? $y = \cos x, \ y' = -\sin x$

Remembering the Properties of Trigonometric Functions

The derivative rules that we have learned previously still apply to trig functions. You also need to recall the things that you knew about trig functions in previous courses.

How can you express y = tan x in terms of sin x and cos x?

$$y = \tan x$$
$$= \frac{\sin x}{\cos x}$$

What are the reciprocal trigonometric ratios?

$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$

What is the Pythagorean Identity?

$$\sin^2 x + \cos^2 x = 1$$

For the graph y = -2 sin (2x + π) - 3, state the period, equation of the axis, amplitude, phase shift, domain and range.

$y = -2\sin\left[2\left(x + \frac{\pi}{2}\right)\right] - 3$

Period $= \frac{2\pi}{2}$
$= \pi$

Axis: $y = -3$

Amp: 2

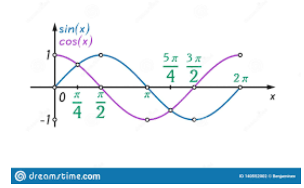
Phase shift: $\frac{\pi}{2}$ units left

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid -5 \leq y \leq -1\}$

How many solutions do trigonometric equations have if an interval for the solution is not given?

An infinite #.



Taking Derivatives of Trigonometric Functions

Please use exact values whenever possible when plugging in values for x. Also, please rationalize denominators!

Differentiate each of the following using derivative rules.

→ Power rule
→ Product rule
→ quotient rule
→ Chain rule

1) f(x) = 2 sin x

$f'(x) = 2\cos x$

2) g(x) = cos2x

$g'(x) = (-\sin 2x)(2)$

$= -2\sin 2x$

(Chain rule)

3) h(x) = -3 sin 3x + 2x^2

$h'(x) = (-3\cos(3x))(3) + 4x$

$= -9\cos 3x + 4x$

(Chain rule)

4) y = 4 cos0.5x - 3(x^2 - x)^2

$y' = (-4\sin 0.5x)(0.5) - 6(x^2 - x)(2x-1)$

$= -2\sin 0.5x - 6x(x-1)(2x-1)$

(Chain rule)

chain rule

5) y = $\underbrace{\sin x}_{1^{st}}\underbrace{\cos 2x}_{2^{nd}}$

$y' = \cos x(\cos 2x) + (-\sin 2x)(2)(\sin x)$

$= \cos x(\cos 2x) - 2\sin 2x \cdot \sin x$

$= \cos x(1 - 2\sin^2 x) - 2(2\sin x\cos x)\sin x$

$= \cos x - 2\sin^2 x\cos x - 4\sin^2 x\cos x$

$= \cos x - 6\sin^2 x\cos x$

$\to = \cos x(1 - 6\sin^2 x)$

What do we do if we have sin^{2}f(x) or cos^{2}f(x)? For example, differentiate the Pythagorean Identity.

$f(x) = \sin^2 x + \cos^2 x$

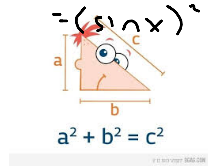
$= (\sin x)^2 + (\cos x)^2$

$f'(x) = 2\sin x\cos x - 2\sin x\cos x$

$= 0$

$\sin^2 x = \sin x \cdot \sin x$

$= (\sin x)^2$



More Practice:

Differentiate each of the following.

1) f(x) = -2 sin^2($\sqrt{x}$)

$= -2(\sin\sqrt{x})^2$

$x^{\frac{1}{2}}$

$\frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = -4(\sin\sqrt{x})(\cos\sqrt{x})(\frac{1}{2}x^{-\frac{1}{2}})$

$= -\frac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

$= \frac{-2\sqrt{x}\sin\sqrt{x}\cos\sqrt{x}}{x}$

2) g(x) = -2(x^2 + 3x - cos^{2}3x)^2

$g'(x) = -4(x^2 + 3x - \cos^2 3x) \cdot (2x + 3 + 6\cos 3x\sin 3x)$

$g'(x) = -4(x^2 + 3x - \cos^2 3x)(2x + 3 + 6\cos 3x\sin 3x)$

Derivative of $y = \cos^2 3x$

$y' = (2\cos 3x)(-\sin 3x)(3)$

$= -6\cos 3x\sin 3x$