Friday, January 31, 2020

1.1 Radical Expressions

You have already been exposed to the idea of rationalizing the denominator. In calculus, we will also have to rationalize the numerator on occasion. We also need to think about "conjugate radicals" now, which are just the same terms separated by opposite signs. We saw this before when we multiplied things like (x - 3) and (x + 3)

What happens when we multiply binomials that are conjugates?

ex/ $(2x + 1)(2x - 1)$

$= 4x^2 - 2x + 2x - 1$

$= 4x^2 - 1$

The middle terms cancel.

Why do you think that this idea might be helpful when we are dealing with radicals?

Multiplying by the conjugate eliminates the radical terms.

ex/ $(\sqrt{3} + 2)(\sqrt{3} - 2)$

$= \sqrt{9} - 2\sqrt{3} + 2\sqrt{3} - 4$

$= 3 - 4$

$= -1$

Why do we have to do this??

- We want to rationalize denominators because division by an integer is easier than division by a radical.
- In calculus, it is also often useful to rationalize a numerator so that we can evaluate limits (this will make more sense next week)

A Review:

Write $\frac{-2}{\sqrt{6}}$ with an integer value in the denominator.

$\frac{-2}{\sqrt{6}} \times \underbrace{\frac{\sqrt{6}}{\sqrt{6}}}_{\text{fancy one}}$

$= \frac{-2\sqrt{6}}{6}$

$= -\frac{\sqrt{6}}{3}$

Extending What We Know:

Rationalize the denominator for $\frac{2\sqrt{3} - 3\sqrt{2}}{\sqrt{2}}$ and $\frac{3\sqrt{5}}{\sqrt{2} - \sqrt{3}}$

$$\frac{2\sqrt{3} - 3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{6} - 3(2)}{2}$$

$$= \frac{2\sqrt{6} - 6}{2}$$

$$= \sqrt{6} - 3$$

$$\frac{3\sqrt{5}}{\sqrt{2} - \sqrt{3}} \times \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}}$$

$$= \frac{3\sqrt{10} + 3\sqrt{15}}{2 + \sqrt{6} - \sqrt{6} - 3}$$

$$= \frac{3\sqrt{10} + 3\sqrt{15}}{-1}$$

$$= -3\sqrt{10} - 3\sqrt{15}$$

Rationalize the numerator for $\frac{2\sqrt{3} - 3\sqrt{2}}{\sqrt{2}}$

$$= \frac{2\sqrt{3} - 3\sqrt{2}}{\sqrt{2}} \times \frac{2\sqrt{3} + 3\sqrt{2}}{2\sqrt{3} + 3\sqrt{2}}$$

$$= \frac{4(3) + \cancel{6\sqrt{6}} - \cancel{6\sqrt{6}} - 9(2)}{2\sqrt{6} + 3(2)}$$

$$= \frac{-6}{2\sqrt{6} + 6}$$

$$= \frac{-3}{\sqrt{6} + 3}$$

What if there are letters??

Do what you know! Rationalize the numerator for $\frac{\sqrt{x - 1} + 3}{x - 2}$

$$\frac{\sqrt{x-1} + 3}{x - 2} \times \frac{\sqrt{x-1} - 3}{\sqrt{x-1} - 3}$$

$$= \frac{x - 1 - 9}{(x-2)(\sqrt{x-1} - 3)}$$

$$= \frac{x - 10}{(x-2)(\sqrt{x-1} - 3)}$$

Please be careful with signs, and do not try to memorize anything - rationalizing radical expressions actually makes sense. If something looks complex, bring it back to simple number to make sure that you are not inventing math!!