

Calculus Appendix

$$2a) \frac{d}{dx} x^2 + \frac{d}{dx} y^2 = \frac{d}{dx} 36 \quad b) \frac{d}{dx} 15y^2 = \frac{d}{dx} 2x^3$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$30y \frac{dy}{dx} = 6x^2$$

$$\frac{dy}{dx} = \frac{x^2}{5y}$$

$$c) 3xy^2 + y^3 = 8$$

$$\frac{d}{dx} 3xy^2 + \frac{d}{dx} y^3 = \frac{d}{dx} 8$$

$$3x \left(2y \frac{dy}{dx} \right) + y^2(3) + 3y^2 \frac{dy}{dx} = 0$$

$$6xy \frac{dy}{dx} + 3y^2 + 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (6xy + 3y^2) = -3y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{2xy + 3y^2} \quad \text{or} \quad \frac{-y}{2x + 3y} ?$$

$$d) 9x^2 - 16y^2 = -144$$

$$18x - 32y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{9x}{16y}$$

$$e) \frac{x^2}{16} + \frac{3y^2}{13} = 1$$

$$\frac{x}{8} + \frac{6y}{13} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{8} \times \frac{13}{6y}$$

$$f) x^2 + y^2 + 5y = 10$$

$$2x + 2y \frac{dy}{dx} + 5 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (2y + 5) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y + 5}$$

$$3a) 2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$m = \frac{2}{3}$$

$$y = mx + b$$

$$-3 = \frac{2}{3}(2) + b$$

$$-\frac{13}{3} = b$$

$$y = \frac{2}{3}x - \frac{13}{3}$$

$$b) x^2 + 4y^2 = 100$$

$$2x + 8y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{4y}$$

$$m = \frac{2}{3}$$

$$3 = \frac{2}{3}(-8) + b$$

$$\frac{25}{3} = b$$

$$y = \frac{2}{3}x + \frac{25}{3}$$

$$c) \frac{2x}{25} - \frac{y}{18} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{36x}{25y}$$

$$m = \frac{180\sqrt{3}}{5}$$

$$= -\frac{300}{5\sqrt{3}}$$

$$y = \frac{-3\sqrt{3}x - 3}{5}$$

$$d) \frac{2x}{81} - \frac{5y}{81} \frac{dy}{dx} = 0$$

$$y = mx + b$$

$$-4 = 11(-11) + b$$

$$\frac{dy}{dx} = \frac{2x}{5y}$$

$$m = \frac{22}{20} \text{ or } \frac{11}{10}$$

$$\frac{10}{81} - b$$

$$y = \frac{11}{10}x + \frac{81}{10}$$

$$4) 1 + 2y \frac{dy}{dx} = 0$$

$$x + 2y = 0$$

$$5a) 5x^2 - 6xy + 5y^2 = 16$$

$$10x - (6x \frac{dy}{dx} + y(-6)) + 10y \frac{dy}{dx} = 0$$

When does $\frac{dy}{dx} = -\frac{1}{2}$

$$\frac{dy}{dx} = -\frac{1}{2y}$$

$$-1 = -\frac{1}{2y}$$

$$y = -\frac{x}{2}$$

$$m = -\frac{1}{2}$$

$$10x - 6y - 6x \frac{dy}{dx} + 10y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (-6x + 10y) = 6y - 10x$$

$$-2y = -2$$

$$y = 1$$

$$x + y^2 = 1$$

$$x + 1 = 1 \quad (0, 1)$$

$$x = 0$$

$$\frac{dy}{dx} = \frac{-2(5x - 3y)}{-2(3x - 5y)}$$

$$\frac{dy}{dx} = \frac{5x - 3y}{3x - 5y}$$

$$\text{At } (1, -1): \frac{dy}{dx} = \frac{8}{8} = 1$$

$$6) 5x^2 + y^2 = 21$$

$$10x + 2y \frac{dy}{dx} = 0$$

$$b) \text{ Let } \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{5x}{y}$$

$$5x - 3y = 0$$

$$x = \frac{3y}{5}$$

$$\text{At } A: \frac{dy}{dx} = -10$$

$$5\left(\frac{3y}{5}\right)^2 - 6\left(\frac{3y}{5}\right)(y) + 5y^2 = 16$$

$$\frac{9y^2}{5} - \frac{18y^2}{5} + 5y^2 = 16$$

$$\frac{16y^2}{5} = 16$$

$$y^2 = 5$$

$$y = \pm \sqrt{5}$$

$$x = \pm \frac{3\sqrt{5}}{5}$$

$$7. \quad x^3 + y^3 - 3xy = 17$$

$$3x^2 + 3y^2 \frac{dy}{dx} - (3x \frac{dy}{dx} + y(3)) = 0$$

$$3x^2 - 3y + (3y^2 - 3x) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}$$

$$= \frac{y - x^2}{y^2 - x}$$

At (2,3): $\frac{dy}{dx} = \frac{3-4}{7}$
 $= -\frac{1}{7}$

normal: $m = 7$

$$y = mx + b$$

$$3 = 7(2) + b$$

$$-11 = b$$

$$y = 7x - 11$$

10. $4x^2y - 3y = x^3$

a) $4x^2 \frac{dy}{dx} + 8xy - 3 \frac{dy}{dx} = 3x^2$

$$\frac{dy}{dx} (4x^2 - 3) = 3x^2 - 8xy$$

$$\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$$

b) $y(4x^2 - 3) = x^3$

$$y = \frac{x^3}{4x^2 - 3}$$

$$y' = \frac{3x^2(4x^2 - 3) - x^3(8x)}{(4x^2 - 3)^2}$$

$$\frac{dy}{dx} = \frac{12x^4 - 9x^2 - 8x^4}{(4x^2 - 3)^2}$$

$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$

8. $2y \frac{dy}{dx} = \frac{3x^2(2-x) - x^3(-1)}{(2-x)^2}$

$$\frac{dy}{dx} = \frac{6x^2 - 3x^3 + x^3}{2y(2-x)^2}$$

$$= \frac{3x^2 - x^3}{y(2-x)^2}$$

When at (1, -1): $\frac{dy}{dx} = \frac{2}{-1} = -2$

normal: $m = \frac{1}{2}$

$$y = mx + b$$

$$-1 = \frac{1}{2}(1) + b$$

$$-\frac{3}{2} = b \quad y = \frac{1}{2}x - \frac{3}{2}$$

9a) $3(x+y)^2 (1 + \frac{dy}{dx}) = 12$

$$\frac{dy}{dx} = \frac{4}{(x+y)^2} - 1$$

b) $(x+y)^{1/2} - 2x = 1$

$$\frac{1}{2}(x+y)^{-1/2} (1 + \frac{dy}{dx}) - 2 = 0$$

$$1 + \frac{dy}{dx} = 2$$

$$2(x+y)^{1/2}$$

$$\frac{dy}{dx} = \frac{4\sqrt{x+y}}{2} - 1$$

c) $\frac{dy}{dx} = \frac{3x^2 - 8x(\frac{x^3}{4x^2 - 3})}{4x^2 - 3}$

$$= \frac{3x^2(4x^2 - 3) - 8x^4}{(4x^2 - 3)^2}$$

$$= \frac{12x^4 - 9x^2 - 8x^4}{(4x^2 - 3)^2}$$

$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$

$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$