Part A

1.a) $3x^2 - y^2 = 23$ Pt (4, 5)

i) $y = \sqrt{3x^2 - 23}$

$\frac{dy}{dx} = \frac{1}{2}(3x^2-23)^{-1/2}(6x)$

$= \frac{3x}{\sqrt{(3x^2-23)}}$

$y' = \frac{12}{5}$

ii) $3x^2 - y^2 = 23$

$6x - 2yy' = 0$

$y' = \frac{3x}{y}$

$y' = \frac{12}{5}$

b) implicit is more efficient.

2a) $x^2 + y^2 = 25$

$2x + 2yy' = 0$

$y' = -\frac{x}{y}$

b) $x^3 + x^2y + 4y^2 = 6$

$3x^2 + 2xy + x^2y' + 8yy' = 0$

$y'(x^2 + 8y) = -3x^2 - 2xy$

$y' = \frac{-3x^2 - 2xy}{x^2 + 8y}$

c) $x^3 + y^3 = 17$

$3x^2 + 3y^2y' = 0$

$y' = \frac{-x^2}{y^2}$

d) $4x^2 - y^2 = 36$

$8x - 2yy' = 0$

$y' = \frac{4x}{y}$

f) $xy = 9$

$y + xy' = 0$

$y' = -\frac{y}{x}$

g) $y^5 + x^2y^3 = 1 + x^4y$

$5y^4y' + 2xy^3 + 3y^2y'x^2 = 4x^3y$

$y'(5y^4 + 3y^2 - x^4) = 4x^3y - 2$

$y' = \frac{2xy(2x^2 - y}{5y^4 + 3y^2 - x}$

e) $y^3 + y = 3x$

$3y^2y' + y' = 3$

$y'(3y^2 + 1) = 3$

$y' = \frac{3}{3y^2 + 1}$

h) $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 9$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$

$\frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} = 0$

$y' = \frac{2\sqrt{y}}{2\sqrt{x}}$

$y' = \sqrt{\frac{y}{x}}$

3a) $x^2 + 9y^2 = 37 \quad (1,2)$

$2x + 18yy' = 0$

$y' = -\frac{x}{9y}$

$y' = -\frac{1}{18}$

$y = mx + b$

$2 = -\frac{1}{18}(1) + b$

$\frac{37}{18} = b$

$y = -\frac{1}{18}x + \frac{37}{18}$

b) $xy = 36, \quad (9,4)$

$y + xy' = 0$

$y' = -\frac{y}{x}$

$y' = -\frac{4}{9}$

$y = mx + b$

$4 = -\frac{4}{9}(9) + b$

$8 = b$

$y = -\frac{4}{9}x + 8$

c) $x^2y^2 + xy = 30, \quad (-3,2)$

$2xy^2 + 2yy'x^2 + y + y'x = 0$

$y'(2x^2y + x) = -2xy^2 - y$

$y' = -\frac{y(2xy+1)}{x(2xy+1)}$

$y' = \frac{2}{3}$

$y = mx + b$

$2 = \frac{2}{3}(-3) + b$

$4 = b$

$y = \frac{2}{3}x + 4$

d) $y^4 + x^2y^3 = 5, \quad (2,1)$

$4y^3y' + 2xy^3 + 3y^2y'x^2 = 0$

$y'(4y^3 + 3y^2x^2) = -2xy^3$

$y' = \frac{-2xy^3}{y^2(4y + 3x^2)}$

$y' = \frac{-2(2)(1)}{4(1) + 3(2)^2}$

$= \frac{-4}{16}$

$= -\frac{1}{4}$

$y = mx + b$

$1 = -\frac{1}{4}(2) + b$

$\frac{3}{2} = b$

$y = -\frac{1}{4}x + \frac{3}{2}$

Part B

4a) $(x-3)^2 + (y+1)^2 = 16, \quad (3,-5)$

$2(x-3) + 2(y+1)y' = 0$

$y' = -\frac{(x+3)}{y+1}$

$= \frac{0}{-4}$

$= \frac{3}{2}$

$y = mx + b$

$-5 = 0(3) + b$

$-5 = b$

$\boxed{y = -5}$

b) $y^2 + 2xy = 11, \quad (5,-1)$

$2yy' - (2y + 2xy') = 0$

$2yy' - 2xy' = 2y$

$y'(2y - 2x) = 2y$

$y' = \frac{y}{y-x}$

$y' = \frac{-1}{-6}$

$= \frac{1}{6}$

$y = mx + b$

$-1 = \frac{1}{6}(5) + b$

$-\frac{11}{6} = b$

$y = \frac{1}{6}x - \frac{11}{6}$

4c) $x^2+y^2-4x+6y=87$, $(12,-3)$

$2x+2yy'-4+6y'=0$

$y'(2y+6) = -2x+4$

$y' = \frac{-2(x-2)}{2(y+3)}$

$y' = \frac{-(x-2)}{y+3}$

$= \frac{-10}{0}$

undefined!

Vertical tangent: $x=12$

5a) $x^2+y^2-6x+2y=15$, $(6,3)$

$2x+2yy'-6+2y'=0$

$y'(2y+2) = -2x+6$

$y' = \frac{-x+3}{y+1}$

$y' = \frac{-3}{4}$

$y = mx+b$

$3 = -\frac{3}{4}(6)+b$

$\frac{15}{2} = b$

$y = -\frac{3}{4}x + \frac{15}{2}$

d) $x^2-xy+y^3=3$, $(-1,1)$

$2x-(y+xy')+3y^2y'=0$

$2x-y-xy'+3y^2y'=0$

$y'(3y^2-x) = y-2x$

$y' = \frac{y-2x}{3y^2-x}$

$= \frac{3}{4}$

$y = mx+b$

$1 = \frac{3}{4}(-1)+b$

$\frac{7}{4} = b$

$y = \frac{3}{4}x + \frac{7}{4}$

11. $x^2y^2+xy=2$

$2xy^2+2yy'x^2+y+xy'=0$

$y'(2x^2y+x) = -2xy^2-y$

$y' = \frac{-y(2xy+1)}{x(2xy+1)}$

$= -\frac{y}{x}$

let $y' = -1$

$-1 = -\frac{y}{x}$

$x = y$

When $x=1$, t.s. is -1.

Sub into equation!

$x^2(x^2)+x(x)=2$

$x^4+x^2=2$

$x^4+x^2-2=0$

$(x^2+2)(x^2-1)=0$

$x^2+x-2=0$

$(x+2)(x-1)=0$

inadmissable

$x=\pm 1$

Points: $(1,1)$, $(-1,-1)$