

MCV 4U Handout
Calculus Appendices Review

Derivatives of the Natural Logarithm

Differentiate each of the following. Remember to apply derivative rules as you always have!

1. $f(x) = \cos 2x - 3 \ln x$
2. $y = \ln 2x - 2^x$
3. $y = \frac{\ln x}{x^2 - 3x}$
4. $f(x) = (3 \ln x^2)(\sqrt{x + 2})$
5. $y = \ln(3x - 4x^3) + \sin(2x - 5) + \frac{3}{x} - e^x$

Implicit Differentiation and Related Rates

12. Find a formula for the slope of the tangent at any point (x, y) on each curve.

- a) $x^2 + y^2 = 25$ b) $x^3 y^2 + 4x^2 y = 12$
c) $x^2 y^3 + 2xy = 20$ d) $2y^3 + x^2 y^2 = 29$

8. Find an equation of the tangent to each curve at the given point.

- a) $(x + 2)^2 + (y - 3)^2 = 2$, $(-1, 2)$
b) $2x^2 y^3 + 3xy + 5 = 0$, $(1, -1)$

14. Given $V = \frac{1}{3}\pi r^2 h$ and $r = h$, find $\frac{dr}{dt}$ if $\frac{dV}{dt} = 4$ when $r = h = 6$.

15. A spotlight on the ground shines on the outside wall of a parking garage 12 m away. If a 2-m tall man walks toward the garage at a speed of 0.75 m/s, how fast is the height of the man's shadow on the garage wall decreasing when he is 4 m from the building?

9. A comet passing near the sun “evaporates,” and the evaporated material forms the tail of the comet. Assume that the comet always maintains a spherical shape and that its surface area is decreasing at $250 \text{ m}^2/\text{min}$. Find the rate at which the radius decreases when the radius is 5 km.

10. A water tank at a filtration plant is built in the shape of a circular cone with height 4 m and diameter 5 m at the top. Water is being pumped into the tank at a rate of $1.2 \text{ m}^3/\text{min}$. Find the rate at which the water level is rising when the water is 3 m deep.

17. After a fun morning in the snow, Emily props her 1.5 m aluminum sled up against the house and goes in for lunch. When the bottom of the sled is 1.0 m from the wall, it is slipping farther away from the wall at 15 cm/s . How fast is the top of the sled moving down the wall?