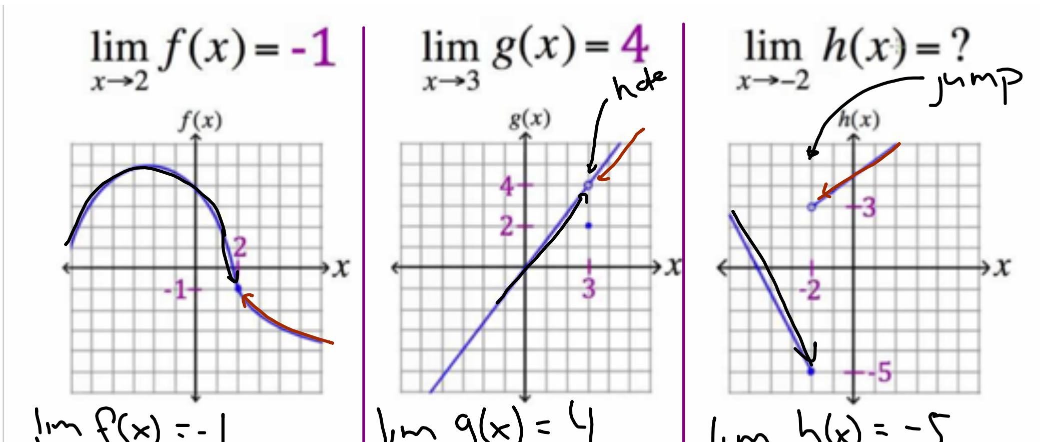
Thursday, February 6, 2020

1.4 The Limit of a Function

Sometimes we are looking for the limiting value for a function rather than the tangent slope to find rate of change. We state this using the notation:

$$\lim_{x \to a} f(x) = L$$

This states that the limit of f(x) as x approaches a equals L. This limit only exists if the value of L is the same when you approach a from the right and the left. This is much easier to see on a graph:



$\lim_{x \to 2^-} f(x) = -1$

$\lim_{x \to 2^+} f(x) = -1$

$\therefore \lim_{x \to 2} f(x) = -1$

$\lim_{x \to 3^-} g(x) = 4$

$\lim_{x \to 3^+} g(x) = 4$

$\lim_{x \to 3} g(x) = 4$

$\lim_{x \to -2^-} h(x) = -5$

$\lim_{x \to -2^+} h(x) = 3$

When $\lim_{x \to -2^-} h(x) \neq \lim_{x \to -2^+} h(x)$

$\lim_{x \to -2} h(x)$ does not exist.

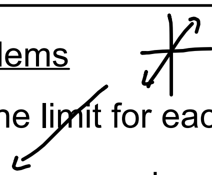
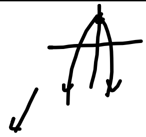
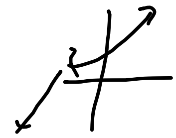
To evaluate limits of functions (remember that we are getting very close to 'a', but not reaching 'a'), we can create a table of values using values that approach 'a', we can graph the function and look at it, or we can use substitution (plug 'a' in to the equation).

Notation Note:

- When we approach a limiting value from the positive side (x is decreasing) we use: $$\lim_{x \to a^+} f(x) = L$$
- When we approach a limiting value from the negative side (x is increasing), we use: $$\lim_{x \to a^-} f(x) = L$$

Practice Problems

1) Calculate the limit for each function.

a. $\lim\limits_{x\to 4} x = 4$

b. $\lim\limits_{x\to -1} (2-5x^2)$

$= 2 - 5(-1)^2$

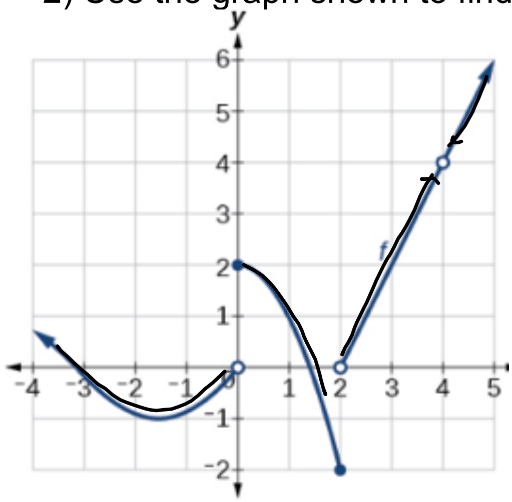
$= -3$

c. $\lim\limits_{x\to 3} 3^x$

$= 3^3$

$= 27$

2) Use the graph shown to find the limits, if they exist.



a. $\lim\limits_{x\to 0^-} f(x) = 0$ b. $\lim\limits_{x\to 0^+} f(x) = 2$ c. $\lim\limits_{x\to 0} f(x)$

Does not exist!

d. $\lim\limits_{x\to 4^-} f(x) = 4$ e. $\lim\limits_{x\to 4^+} f(x) = 4$ f. $\lim\limits_{x\to 4} f(x) = 4$

3) Sketch the graph of the piecewise function given below, and then determine the indicated limit, if it exists.

$$f(x) = \begin{cases} x, if\ x \le -2 \\ x^2 - 3, if\ -2 < x \le 1 \\ -2x + 4, if\ x > 1 \end{cases};\ \lim_{x\to -2} f(x)$$

$\lim\limits_{x\to -2^+} f(x) = 1$

$\lim\limits_{x\to -2^-} f(x) = -2$

$\therefore \lim\limits_{x\to 2} f(x)$ does not exist.