

Thursday, February 6, 2020

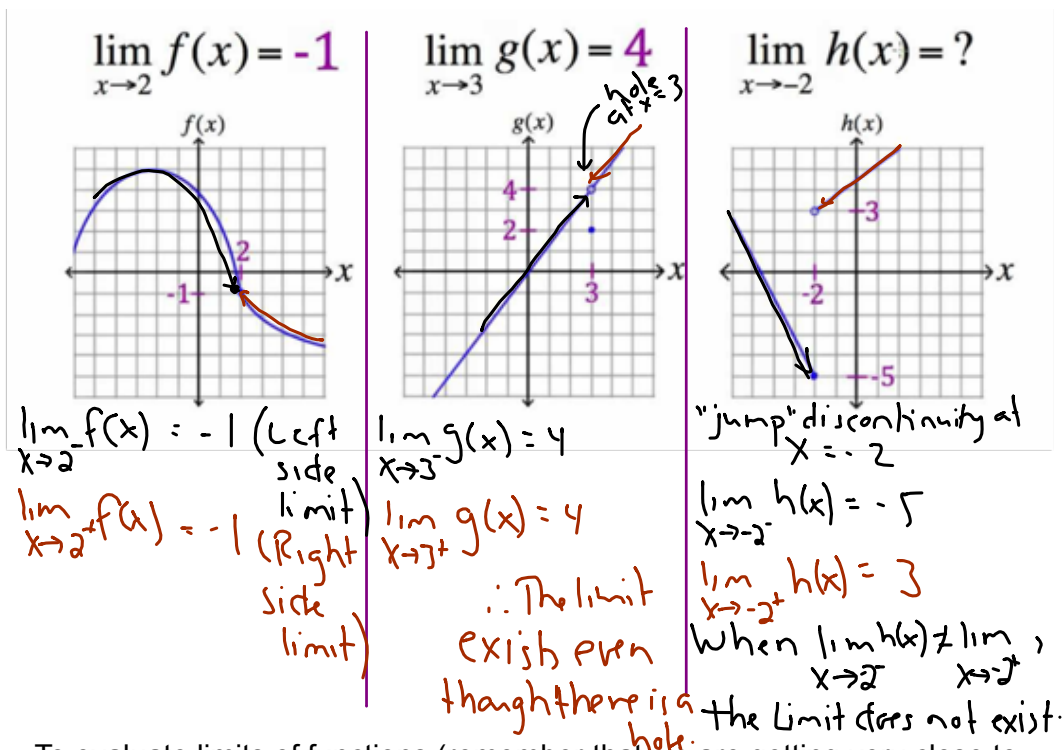


## 1.4 The Limit of a Function

Sometimes we are looking for the limiting value for a function rather than the tangent slope to find rate of change. We state this using the notation:

$$\lim_{x \rightarrow a} f(x) = L$$

This states that the limit of  $f(x)$  as  $x$  approaches  $a$  equals  $L$ . This limit only exists if the value of  $L$  is the same when you approach  $a$  from the right and the left. This is much easier to see on a graph:



To evaluate limits of functions (remember that we are getting very close to 'a', but not reaching 'a'), we can create a table of values using values that approach 'a', we can graph the function and look at it, or we can use substitution (plug 'a' in to the equation).

Notation Note:

- When we approach a limiting value from the positive side ( $x$  is decreasing) we use:

$$\lim_{x \rightarrow a^+} f(x) = L$$

- When we approach a limiting value from the negative side ( $x$  is increasing), we use:

$$\lim_{x \rightarrow a^-} f(x) = L$$

## Practice Problems

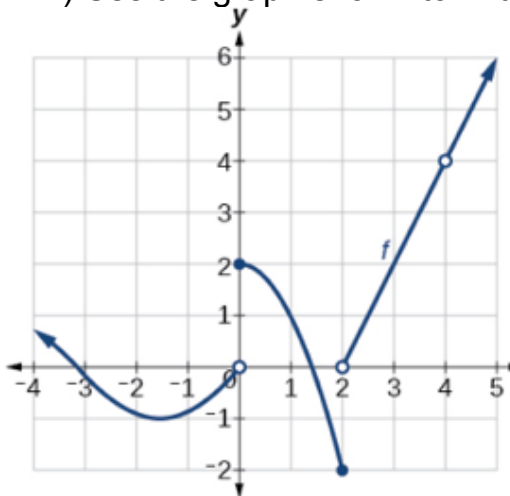
1) Calculate the limit for each function.

a.  $\lim_{x \rightarrow 4} x = 4$

b.  $\lim_{x \rightarrow -1} (2 - 5x^2)$   
 $\sim 2 - 5(-1)^2$   
 $= -3$

c.  $\lim_{x \rightarrow 3} 3^x$   
 $= 3^3$   
 $= 27$

2) Use the graph shown to find the limits, if they exist.



a.  $\lim_{x \rightarrow 0^-} f(x) = 0$

b.  $\lim_{x \rightarrow 0^+} f(x) = 2$

c.  $\lim_{x \rightarrow 0} f(x)$

Does not exist

d.  $\lim_{x \rightarrow 4^-} f(x) = 4$

e.  $\lim_{x \rightarrow 4^+} f(x) = 4$

f.  $\lim_{x \rightarrow 4} f(x)$

$= 4$

3) Sketch the graph of the piecewise function given below, and then determine the indicated limit, if it exists.

$$f(x) = \begin{cases} x, & \text{if } x \leq -2 \\ x^2 - 3, & \text{if } -2 < x \leq 1 \\ -2x + 4, & \text{if } x > 1 \end{cases}; \lim_{x \rightarrow -2} f(x)$$

$\lim_{x \rightarrow -2^-} f(x) = -2$

$\lim_{x \rightarrow -2^+} f(x) = 1$

$\therefore \lim_{x \rightarrow -2} f(x)$  does not exist.

