p. 9 #6b

6b) $\dfrac{2\sqrt{6}}{2\sqrt{27} - \sqrt{8}}$

$= \dfrac{2\sqrt{6}}{2\sqrt{9\times 3} - \sqrt{4\times 2}}$

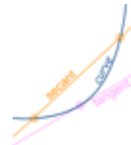
$= \dfrac{2\sqrt{6}}{6\sqrt{3} - 2\sqrt{2}}$

$= \dfrac{\sqrt{6}}{3\sqrt{3} - \sqrt{2}} \times \dfrac{3\sqrt{3} + \sqrt{2}}{3\sqrt{3} + \sqrt{2}}$

$= \dfrac{3\sqrt{18} + \sqrt{12}}{9(3) - 2}$

$= \dfrac{3(3\sqrt{2}) + 2\sqrt{3}}{25}$

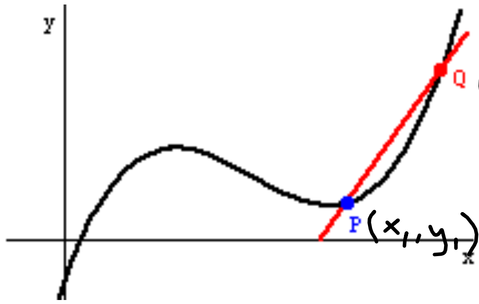
$= \dfrac{9\sqrt{2} + 2\sqrt{3}}{25}$



Monday, February 3, 2020

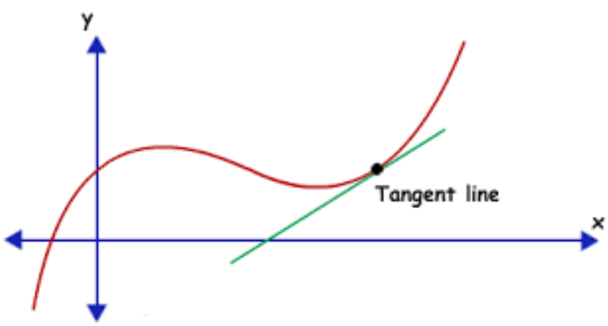
Review of Rate of Change

average rate of change - the slope of a line segment that joins two points on a function (secant line)

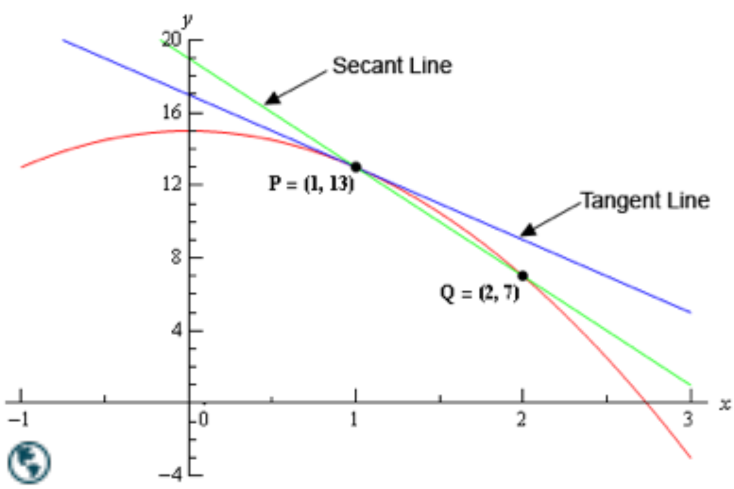


$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

instantaneous rate of change - the slope of a line segment that touches a function at one specific point (tangent line)



When we move the points that a secant passes through closer and closer together, the slope of the secant (average rate of change) approaches the slope of the tangent (instantaneous rate of change).



Because we could not evaluate the slope of a tangent line directly, we had to estimate instantaneous rate of change. To do this, we used the difference quotient (STILL THE SLOPE FORMULA!!).

$$\text{i.r.o.c} = \frac{f(a+h)-f(a)}{a+h-a}$$

, where (a, f(a)) and (a + h, f(a + h)) are two points on a function and h is a very small number.

Let's try it! Estimate the instantaneous rate of change of the function $f(x) = 2x^2 - 3x + 4$ when $x = 2$.

Advanced Functions: Let h = 0.01.

$f(2) = 6$

$f(2.01) = 6.0502$

$$\text{iroc} = \frac{f(2.01)-f(2)}{2.01-2} = \frac{6.0502-6}{0.01} = 5.02$$

$\therefore$ The i.r.o.c. is ~5.

Calculus: Work with h and simplify.

$f(2) = 6$

$$\begin{aligned} f(2+h) &= 2(2+h)^2 - 3(2+h) + 4 \\ &= 2(4+4h+h^2) - 6 - 3h + 4 \\ &= 8 + 8h + 2h^2 - 6 - 3h + 4 \\ &= 2h^2 + 5h + 6 \end{aligned}$$

$$\text{iroc} = \frac{2h^2+5h+6-6}{2+h-2} = \frac{2h^2+5h}{h} = 2h+5$$

Let $h=0$

$\therefore$ iroc is 5.

What does the sign of the instantaneous rate of change tell us about the function at that point?

$\oplus \rightarrow$ function is increasing

$\ominus \rightarrow$ function is decreasing

iroc $= 0 \rightarrow$ turning point

How could I find the equation of the tangent line for the example above?

Use the slope ($m=5$) and a point $(2, 6)$ and sub into $y = mx + b$ to solve for b.

Point of tangency

$6 = 5(2) + b$

$-4 = b$

$y = 5x - 4$

1.2 The Slope of a Tangent

Calculus is a branch of mathematics that is concerned with the slope of a tangent (rate of change at a point = differential calculus) and the area under a curve between x = a and x = b (integral calculus).

We discuss slopes of tangents and instantaneous rates of change in this course. Integral calculus comes later...

What Changes?

We just saw that the slope of a secant approaches the slope of a tangent at P(a, f(a)) as the points on the secant P(a, f(a)) and Q(b, f(b)) get closer together.

This means that the secant slope APPROACHES the tangent slope. Another way to think of this is that the slope of the tangent is the LIMIT of the slope of the secant as Q approaches P.

Think about 'h' in our difference quotient. It's value gets smaller and smaller as we approach our point of tangency. What value is h approaching?

h approaches zero.

Why can't we just let h = 0 when we use the difference quotient? (Try it)

$$\text{i.r.o.c} = \frac{f(a+h) - f(a)}{a + h - a}$$

$h=0$

$$= \frac{f(a+0) - f(a)}{a+0-a}$$

$$= \frac{f(a) - f(a)}{a-a}$$

$$= \frac{0}{0} \Big\} \text{indeterminate form}$$

Resolving the Problem of Zero Over Zero - Slope as a Limit!

If we simplify the difference quotient for a function at a point, and leave 'h' in our expression, we can plug h = 0 in at the end to find the exact value of the tangent slope! Let's try it by finding the slope of the tangent to y = $2x^2$ at the point (-2, 8).

$$m = \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h}$$
$$= \lim_{h\to 0} \frac{2(-2+h)^2 - 8}{h}$$
$$= \lim_{h\to 0} \frac{2(4-4h+h^2)-8}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{8} - 8h + 2h^2 - \cancel{8}}{h}$$

$$m = \lim_{h\to 0}(-8 + 2h)$$

Let h = 0

$$\boxed{m = -8}$$

This is the i.r.o.c., not an estimate of.

Notes About Notation:

To find the tangent slope using limits, we write the following:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This is the difference quotient, and the first part is simply the notation to state "the limit as h approaches zero". Instead of m, you can also use f ' (x), which is stated as "f prime at x". We will use this notation a lot with derivatives, so it is okay to start thinking about it now.

Example: Determine the slope of the tangent to the curve represented by $f(x) = \sqrt{25 - x^2}$ at the point (3, 4).

$$f'(x) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{25-(9+6h+h^2)} - 4}{h}$$

RATIONALIZE THE NUMERATOR!

$$= \lim_{h\to 0} \frac{\sqrt{16-6h-h^2} - 4}{h} \times \frac{\sqrt{16-6h-h^2} + 4}{\sqrt{16-6h-h^2} + 4}$$

$$= \lim_{h\to 0} \frac{\cancel{16} - 6\cancel{h} - h^{\cancel{2}} - \cancel{16}}{\cancel{h}\left(\sqrt{16-6h-h^2} + 4\right)}$$
$$= \lim_{h\to 0} \frac{-6-h}{\sqrt{16-6h-h^2} + 4}$$

Let h = 0

$$f'(x) = \frac{-6}{8}$$
$$= -\frac{3}{4}$$

∴ The slope of the tangent is $-\frac{3}{4}$.