Wednesday, February 26, 2020

2.4 The Quotient Rule

Yesterday you had to rewrite rational functions as products raised to negative powers to take their derivative. There is a rule that allows us to work directly from rational form.

The Quotient Rule

In words : The derivative of the top times the bottom minus the top times the derivative of the bottom, all over the bottom squared.

Algebraically: If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Try it! Differentiate $h(x) = \dfrac{2x-3}{x^2+2x}$

$$h'(x) = \frac{2(x^2+2x) - (2x-3)(2x+2)}{(x^2+2x)^2}$$

$$= \frac{2x^2+4x - (4x^2-2x-6)}{(x^2+2x)^2}$$

$$= \frac{-2x^2+6x+6}{(x^2+2x)^2}$$

It is important to note that order matters in the Quotient Rule!

Proof: We will use the product rule to prove the quotient rule.

① Rewrite as a product. $h(x) = \dfrac{f(x)}{g(x)} \rightarrow h(x) = f(x)[g(x)]^{-1}$

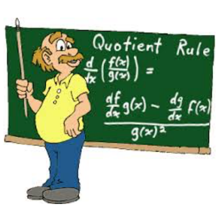
② Apply the product rule.

$$h'(x) = f'(x)[g(x)]^{-1} + \left[-[g(x)]^{-2}g'(x)\right]f(x)$$

$$= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{[g(x)]^2}$$

Get a CD → by multiplying the first term by $\frac{g(x)}{g(x)}$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$



Practice Problems:

1. Use the quotient rule to differentiate $h(x) = \frac{x^3+2x-3}{x^2+5}$. Simplify your answer.

$$h'(x) = \frac{(3x^2+2)(x^2+5) - (2x)(x^3+2x-3)}{(x^2+5)^2}$$

$$= \frac{3x^4 + 17x^2 + 10 - 2x^4 - 4x^2 + 6x}{(x^2+5)^2}$$

$$= \frac{x^4 + 13x^2 + 6x + 10}{(x^2+5)^2}$$

2. Determine the coordinates of each point on the graph of $h(x) = \frac{2x+8}{\sqrt{x}}$ where the tangent is horizontal.

When does $h'(x) = 0$?

$$h'(x) = \frac{2(x^{1/2}) - (2x+8)(\frac{1}{2}x^{-1/2})}{x}$$

$$= \frac{2x^{1/2} - x^{1/2} - 4x^{-1/2}}{x}$$

$$= \frac{x^{1/2} - 4x^{-1/2}}{x}$$

$$\cancel{x}\left[\frac{x^{1/2} - 4x^{-1/2}}{\cancel{x}}\right] = (0)x$$

$$x^{1/2}\left[x^{1/2} - \frac{4}{x^{1/2}}\right] = [0]x^{1/2}$$

$$x - 4 = 0$$

$$\boxed{x = 4}$$

$$h(4) = \frac{16}{2} = 8$$

$(4, 8)$

3. Determine the equation of the tangent to $y = \frac{2x}{x^2+1}$ when x = -1.

$$y' = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$$

$$= \frac{-2x^2+2}{(x^2+1)^2}$$

$$= \frac{-2(x^2-1)}{(x^2+1)^2}$$

When $x = -1$:

$$y' = \frac{-2(1-1)}{4} = 0$$

$$y = \frac{2(-1)}{1+1} = -\frac{2}{2} = -1$$

Pt of tangency is $(-1, -1)$

Equation of horizontal tangent is $y = -1$.

QUOTIENT RULE:

IF $h(x) = \frac{f(x)}{g(x)}$

$$h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$$