Tuesday, February 11, 2020

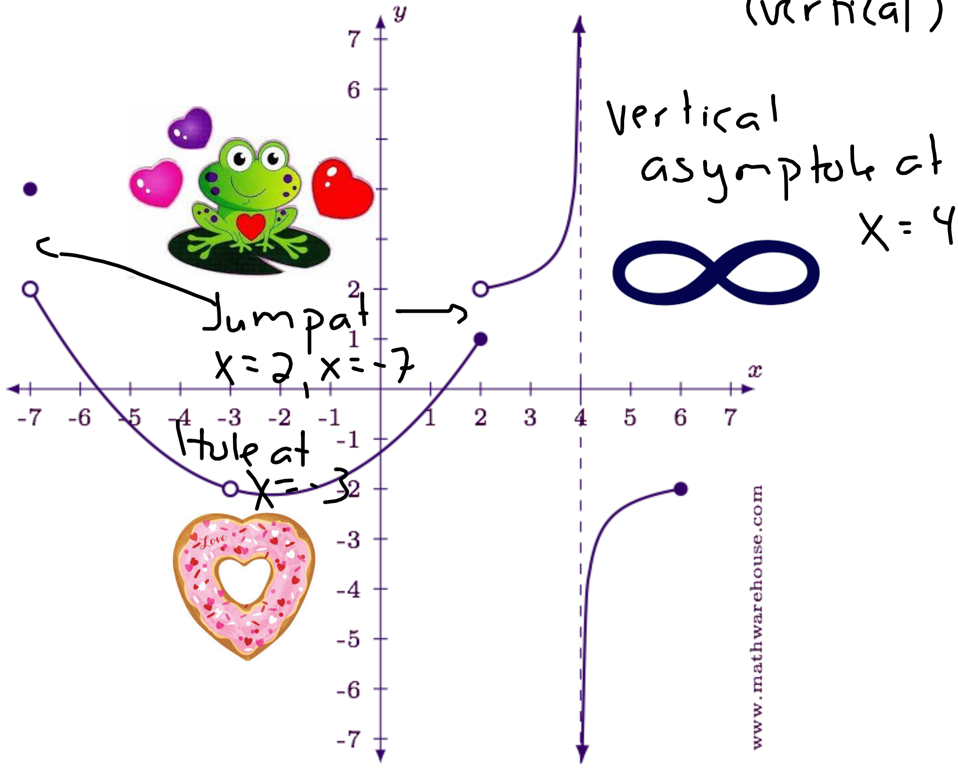
1.6 Continuity

Continuity was discussed in Advanced Functions, and is no different here. We have also indirectly discussed it while we were looking at the existence of a limit at a given value of x.

In simple terms, a function is continuous if you can draw its graph without lifting your pencil from the paper.

Types of Discontinuities (name them, then locate them on the graph)

1) hole 2) jump 3) asymptote (vertical)



For a function to be continuous at a point, x = a, the limit must exist as x approaches a, **and the function must be defined at x = a**. The limit of a function exists at a hole, but the function is not defined at the hole itself, so the function is not continuous at that value!

Reasoning about Continuity

We do not always want to draw a graph, and sometimes even if we wanted to it would be tedious and difficult to do. To determine whether or not a function is continuous at a value of a, we can:

- Reason about the family of functions it comes from (ex/ all polynomial functions are continuous, rational functions have holes or asymptotes, etc.)
- Evaluate the left and right side limits to see if the limit exists, use direct substitution to ensure that the function is defined at 'a' (piecewise functions)

To check for continuity in piecewise functions, you need to check if they intersect at the endpoint of the interval of the domain that they are defined over.

Example: For the function provided, determine if f(x) is continuous at x = -1 and x = 3. Is the function continuous on its domain? State the location and type for any discontinuities.

$$f(x) = \begin{cases} ① \; x+3, if\ x < -1 \\ ② \; \frac{1}{x} + 3, if\ -1 \le x \le 3 \\ ③ \; (x-4)^2, if\ x > 3 \end{cases}$$

① $f(x) = x+3$
$f(-1) = 2$

② $f(x) = \frac{1}{x}+3$
$f(-1) = 2$

$\therefore$ The graph is continuous at $x=-1$.

② $f(3) = \frac{1}{3}+3$
$= 3\frac{1}{3}$

③ $f(3) = (3-4)^2$
$= 1$

$\therefore$ There is a jump discontinuity at $x=3$.

When $x=0$ there is a vertical asymptote as well.

Example: Find k if g(x) is continuous.

$$g(x) = \begin{cases} \sqrt{k} - 4, if\ x \le 4 \\ x - 2, if\ x > 4 \end{cases}$$

To be continuous

$\sqrt{k} - 4 = x - 2$ at $x = 4$

$\sqrt{k} - 4 = 2$

$\sqrt{k} = 6$

$k = 36$

Try not to overcomplicate continuity - you knew how to do this last month, before you knew anything about limits. Nothing is different!