Wednesday, February 26, 2020

2.4 The Quotient Rule

Yesterday you had to rewrite rational functions as products raised to negative powers to take their derivative. There is a rule that allows us to work directly from rational form.

The Quotient Rule

In words : The derivative of the top times the bottom minus the top times the derivative of the bottom, all over the bottom squared.

Algebraically: If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Try it! Differentiate $h(x) = \dfrac{2x-3}{x^2+2x}$

$$h'(x) = \frac{2(x^2+2x) - (2x+2)(2x-3)}{(x^2+2x)^2}$$

$$= \frac{2x^2+4x-(4x^2-2x-6)}{(x^2+2x)^2}$$

$$= \frac{-2x^2+6x+6}{(x^2+2x)^2}$$

It is important to note that order matters in the Quotient Rule!

Proof: We will use the product rule to prove the quotient rule.

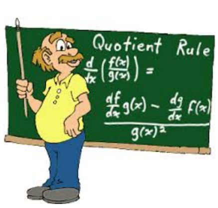
① Rewrite $h(x)$ as a product of 2 functions. $h(x) = \dfrac{f(x)}{g(x)} \rightarrow h(x) = f(x)\cdot\dfrac{1}{g(x)}$

(or $h(x)g(x) = f(x)$) $\quad h(x) = f(x)[g(x)]^{-1}$

② Apply the product rule. $h'(x) = f'(x)[g(x)]^{-1} + [-[g(x)]^{-2}\cdot g'(x)]f(x)$

③ Get a common denominator. $= \dfrac{f'(x)}{g(x)} - \dfrac{f(x)g'(x)}{[g(x)]^2}$

Quotient Rule → $h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$:)



Practice Problems:

1. Use the quotient rule to differentiate $h(x) = \dfrac{x^3 + 2x - 3}{x^2 + 5}$. Simplify your answer.

$$h'(x) = \frac{(3x^2+2)(x^2+5) - (2x)(x^3+2x-3)}{(x^2+5)^2}$$

$$= \frac{3x^4 + 15x^2 + 2x^2 + 10 - 2x^4 - 4x^2 + 6x}{(x^2+5)^2}$$

$$= \frac{x^4 + 13x^2 + 6x + 10}{(x^2+5)^2}$$

2. Determine the coordinates of each point on the graph of $h(x) = \dfrac{2x+8}{\sqrt{x}}$ where the tangent is horizontal.

Slope is zero.

When is $h'(x) = 0$?

$f(x) = 2x + 8$

$f'(x) = 2$

$g(x) = \sqrt{x}$

$g'(x) = \frac{1}{2}x^{-1/2}$

$$h'(x) = \frac{2x^{1/2} - \frac{1}{2}x^{-1/2}(2x+8)}{x}$$

$$0 = \frac{2x^{1/2} - x^{1/2} - 4x^{-1/2}}{x}$$

$$x(0) = \cancel{x}\left(\frac{x^{1/2} - 4x^{-1/2}}{\cancel{x}}\right)$$

$$x^{1/2}(0) = x^{1/2}\left[x^{1/2} - \frac{4}{x^{1/2}}\right]$$

$$0 = x - 4$$

$$4 = x$$

$h(4) = \frac{16}{2}$

$= 8$

$(4, 8)$

3. Determine the equation of the tangent to $y = \dfrac{2x}{x^2+1}$ when x = -1.

$$y' = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$$

$$y' = \frac{-2x^2 + 2}{(x^2+1)^2}$$

When $x = -1$

$$y' = \frac{-2+2}{4}$$

$y' = 0$ ← tangent is horizontal

When $x = -1$

$$y = \frac{-2}{2}$$

$y = -1$

Tangent:

$y = -1$

QUOTIENT RULE:

IF $h(x) = \dfrac{f(x)}{g(x)}$

$$h'(x) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g(x)^2}$$