Vector Appendix – Gaussian Elimination

1a) $\left[\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ -1 & 3 & -2 & -1 \\ 0 & 3 & -2 & -3 \end{array}\right]$ b) $\left[\begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 0 & 2 & -1 & 16 \\ -3 & 1 & 0 & 10 \end{array}\right]$ c) $\left[\begin{array}{ccc|c} 2 & -1 & -1 & -2 \\ 1 & -1 & 4 & -1 \\ -1 & -1 & 0 & 13 \end{array}\right]$

2a) $\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 3 & -1 & 1 \end{array}\right] \xrightarrow{3R_1-2R_2\to R_2} \overset{①}{\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 11 & -2 \end{array}\right]} \xrightarrow[R_2\div 2\to R_2]{R_1\div 2\to R_1} \overset{②}{\left[\begin{array}{cc|c} 1 & \frac{3}{2} & 0 \\ 0 & \frac{11}{2} & -1 \end{array}\right]}$

3. $\left[\begin{array}{ccc|c} 2 & 1 & 6 & 0 \\ 0 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right] \xrightarrow{3R_1-2R_3\to R_3} \left[\begin{array}{ccc|c} 2 & 1 & 6 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 1 & 18 & -2 \end{array}\right] \xrightarrow{2R_3+R_2\to R_3} \left[\begin{array}{ccc|c} 2 & 1 & 6 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 37 & -4 \end{array}\right]$

4a) $\left[\begin{array}{ccc|c} -1 & 0 & 1 & 2 \\ 0 & -1 & 2 & 0 \\ \frac{1}{2} & -\frac{3}{4} & -2 & \frac{1}{3} \end{array}\right] \xrightarrow[12R_3\to R_3]{-1R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & -1 & 2 & 0 \\ 6 & -9 & -24 & 4 \end{array}\right] \xrightarrow[-1R_2\to R_2]{6R_1-R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & 0 \\ 0 & 9 & 18 & -16 \end{array}\right]$

$\xrightarrow{9R_2-R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -36 & 16 \end{array}\right]$

b) $z = \frac{-16}{36}$ or $-\frac{4}{9}$

$y - 2z = 0$

$y - 2(-\frac{4}{9}) = 0$

$y = \frac{8}{9}$

$x - z = -2$

$x = -2 - \frac{4}{9}$

$x = -22/9$

5a) $x - 2y = -1$
$2x - 3y = 1$
$2x - y = 0$

b) $-2x - z = 0$
$x - 2y = 4$
$y + 2z = -3$

c) $z = 0$
$x = -2$
$y + z = 0$

6a) $-5y = 15$
$y = -3$
$-2x + y = 6$
$-2x - 3 = 6$
$x = -\frac{9}{2}$
$(-\frac{9}{2}, -3$

6b) $6z = -36$
$z = -6$
$2y + 3z = 0$
$2y + 3(-6) = 0$
$2y = 18$
$y = 9$
$2x - y + z = 11$
$2x - 9 - 6 = 11$
$2x = 26$
$x = 13$

c) $0 = -13$ ← not possible, so no solution

d) $z = -5$
$y = -4$
$4x - y - z = 0$
$4x + 5 + 4 = 0$
$x = \frac{-9}{4}$
$(-\frac{9}{4}, -4, -5)$

e) $z = t$
$y = s$
$x - s + 3t = 2$
$x = 2 + s - 3t$

f) $z = -2$
$y + 2(-2) = 4$
$y = 8$
$x = 4$
$(4, 8, -2)$

Hilary

7a) Row-echelon, because the zero rows is on the bottom and remaining rows are arranged from greatest # of terms to least.

b) From R_3 we can say $z = t$, but in R_2 we have $0 = -3$, and this can never be true.

c) $\left[\begin{array}{ccc|c} -1 & 1 & 1 & 3 \\ -1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 3 \end{array}\right]$

8a) No.

$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_2 - R_3 \Rightarrow R_3} \left[\begin{array}{ccc|c} -1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 \end{array}\right]$$

9a) $z = \frac{1}{2}$

$y = 0$

$x = -\frac{5}{2}$

$(-\frac{5}{2}, 0, \frac{1}{2})$

9b) The point where three planes meet.

8b) No.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 3 & 1 & -4 & 2 \\ 0 & 0 & 3 & 6 \end{array}\right] \xrightarrow{3R_1 - R_2 \Rightarrow R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & -1 & 10 & -11 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

9a) $z = 2$

$y = 31$

$x = -7$

$(-7, 31, 2)$

b) The point where 3 planes meet.

8c) No.

$$\left[\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -6 \end{array}\right] \xrightarrow{R_2 \Leftrightarrow R_3} \left[\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

9a) Let $y = t$

$z = -6$

$x = -6 + 2t$

$(2t - 6, t, -6)$

b) The line that the 3 planes meet along.

8d) Yes.

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

9a) $z = t$

$y = -3 - 2t$

$x = 4s - t$

b) The line at which 3 planes meet.

10a) $$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 9 \\ 1 & -2 & 1 & 15 \\ 2 & -1 & -1 & -12 \end{array}\right] \xrightarrow[2R_1 + R_3 \Rightarrow R_3]{R_1 + R_2 \Rightarrow R_2} \left[\begin{array}{ccc|c} -1 & 1 & 1 & 9 \\ 0 & -1 & 2 & 24 \\ 0 & 1 & 1 & 6 \end{array}\right] \xrightarrow{R_2 + R_3 \Rightarrow R_3} \left[\begin{array}{ccc|c} -1 & 1 & 1 & 9 \\ 0 & -1 & 2 & 24 \\ 0 & 0 & 3 & 30 \end{array}\right]$$

$3z = 30$

$z = 10$

$-y + 2z = 24$

$-y = 4$

$y = -4$

$-x + y + z = 9$

$-x - 4 + 10 = 9$

$-x = 3$

$(-3, -4, 10)$

Scanned with CamScanner

b) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 3 & 1 & 0 \\ -3 & -2 & -4 & 0 \end{array}\right] \xrightarrow[3R_1+R_3\to R_3]{2R_1-R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \xrightarrow{R_2+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$z=t$ $\quad -y+z=0$ $\quad x+y+z=0$
$-y=-t$ $\quad x+2t=0$ $\quad (-2t,t,t)$
$y=t$ $\quad x=-2t$ $\quad$ (line of intersection)

c) $\left[\begin{array}{ccc|c} 1 & -1 & 3 & -1 \\ 5 & 1 & -3 & -5 \\ 2 & 1 & -3 & -2 \end{array}\right] \xrightarrow[2R_1-R_3\to R_3]{5R_1-R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -1 & 3 & -1 \\ 0 & -6 & 18 & 0 \\ 0 & -3 & 9 & 0 \end{array}\right] \xrightarrow{2R_3-R_2\to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 3 & -1 \\ 0 & -6 & 18 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$z=t$ $\quad -6y=-18t$ $\quad x-3t+3t=-1$
$y=3t$ $\quad x=-1$ $\quad (-1,3t,t)$
(line of intersection)

d) $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 4 \\ -1 & 3 & 8 & -4 \\ 1 & -3 & -4 & -4 \end{array}\right] \xrightarrow[R_1-R_3\to R_3]{R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 4 \\ 0 & 6 & 12 & 0 \\ 0 & 6 & 8 & 8 \end{array}\right] \xrightarrow{R_2-R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 4 \\ 0 & 6 & 12 & 0 \\ 0 & 0 & 4 & -8 \end{array}\right]$

$z=-2$ $\quad 6y+12(-2)=0$ $\quad x+3(4)+4(-2)=4$
$6y=24$ $\quad x+4=4$ $\quad (0,4,-2)$
$y=4$ $\quad x=0$ $\quad$ (P.O.I.)

e) $\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 4 & 2 & 2 & 2 \\ -2 & 1 & 1 & 3 \end{array}\right] \xrightarrow[R_1+R_3\to R_3]{2R_1-R_2\to R_2} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 4 \end{array}\right] \xrightarrow[R_3\div 2]{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$z=t$ $\quad y=2-t$ $\quad 2x=1-(2-t)-t$
$2x=-1$ $\quad (-\frac{1}{2},2-t,t)$
$x=-\frac{1}{2}$ $\quad$ (line of intersection)

f) $\left[\begin{array}{ccc|c} 1 & -1 & 0 & -500 \\ 0 & 2 & -1 & 3500 \\ 1 & 0 & -1 & 2000 \end{array}\right] \xrightarrow{R_3-R_1\to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & -500 \\ 0 & 2 & -1 & 3500 \\ 0 & 1 & -1 & 2500 \end{array}\right] \xrightarrow{R_2-2R_3\to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & -500 \\ 0 & 2 & -1 & 3500 \\ 0 & 0 & 1 & -1500 \end{array}\right]$

$z=-1500$ $\quad y=1000$ $\quad x=500$ $\quad$ POI @ $(500,1000,-1500)$

Hilroy

11. $\left[\begin{array}{ccc|c} 1 & 2 & -1 & a \\ 1 & -1 & 2 & b \\ 3 & 3 & 1 & c \end{array}\right] \xrightarrow[3R_1-R_3\to R_3]{R_2-R_1\to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & a \\ 0 & -3 & 3 & b-a \\ 0 & 3 & -4 & 3a-c \end{array}\right] \xrightarrow{R_2+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & a \\ 0 & -3 & 3 & b-a \\ 0 & 0 & -1 & 2a+b-c \end{array}\right]$

$-z = 2a+b-c$

$z = -2a-b+c$

$-3y + 3(-2a-b+c) = b-a$

$-3y - 6a - 3b + 3c = b-a$

$-3y = 4b + 5a - 3c$

$y = -\frac{5}{3}a - \frac{4}{3}b + c$

$x + 2(-\frac{5}{3}a - \frac{4}{3}b + c) - (-2a-b+c) = a$

$x - \frac{10}{3}a - \frac{8}{3}b + 2c + 2a + b - c = a$

$x - \frac{4}{3}a - \frac{5}{3}b + c = a$

$x = \frac{7}{3}a + \frac{5}{3}b - c$

12. $A(-1,-7)$

$a - b + c = -7$

$B(2,20)$

$4a + 2b + c = 20$

$C(-3,-5)$

$9a - 3b + c = -5$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -7 \\ 4 & 2 & 1 & 20 \\ 9 & -3 & 1 & -5 \end{array}\right] \xrightarrow[9R_1-R_3\to R_3]{4R_1-R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -7 \\ 0 & -6 & 3 & -48 \\ 0 & -6 & 8 & -58 \end{array}\right]$

$\xrightarrow{R_2-R_3\to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -7 \\ 0 & -6 & 3 & -48 \\ 0 & 0 & -5 & 10 \end{array}\right]$

$c = -2$

$-6b - 6 = -48$

$-6b = -42$

$b = 7$

$a - 7 - 2 = -7$

$a = 2$

$y = 2x^2 + 7x - 2$

Chapter 9: The Relationships Between Points, Lines, and Planes

Review of Prerequisite Skills

1a) $P_0(2,-5)$, $\vec{r}=(10,-12)+t(8,-7)$

$2=10+8t \qquad -5=-12-7t$

$-8=8t \qquad 7=-7t$

$-1=t \qquad -1=t$

$\therefore P_0$ is on the line

b) $P_0(1,2)$, $12x+5y-13\stackrel{?}{=}0$

$12(1)+5(2)-13\stackrel{?}{=}0$

$22-13\stackrel{?}{=}0$

$9\neq 0$

$\therefore P_0$ is not on the line

c) $P_0(7,-3,8)$, $\vec{r}=(1,0,-4)+t(2,-1,4)$

$7=1+2t \qquad -3=-t \qquad 8=-4+4t$

$6=2t \qquad t=3 \qquad 12=4t$

$t=3 \qquad 3=t$

$\therefore P_0$ is on the line

d) $P_0(1,0,5)$, $\vec{r}=(2,1,-2)+t(4,-1,2)$

$1=2+4t \qquad 0=1-t \qquad 5=-2+2t$

$-\frac{1}{4}=t \qquad 1=t \qquad \frac{3}{2}=t$

$\therefore P_0$ is not on the line

2a) $P_1(2,5)$, $P_2(7,3)$

$\vec{P_1P_2}=(5,-2)$

$\vec{r}=(2,5)+t(5,-2)$

$x=2+5t$

$y=5-2t$

b) $P_1(-3,7)$, $P_2(4,-7)$

$\vec{P_1P_2}=(7,-14)$

$\vec{r}=(4,-7)+t(7,-14)$

$x=4+7t$

$y=-7-14t$

c) $P_1(-1,0)$, $P_2(-3,-11)$

$\vec{P_1P_2}=(-2,-11)$

$\vec{r}=(-1,0)+t(-2,-11)$

$x=-1-2t$

$y=-11t$

d) $P_1(1,3,5)$, $P_2(6,-7,0)$

$\vec{P_1P_2}=(5,-10,-5)$

$\vec{d}=(1,-2,-1)$

$\vec{r}=(1,3,5)+t(1,-2,-1)$

$x=1+t$

$y=3-2t$

$z=5-t$

e) $P_1(2,0,-1)$, $P_2(-1,5,2)$

$\vec{P_1P_2}=(-3,5,3)$

$\vec{r}=(2,0,-1)+t(-3,5,3)$

$x=2-3t$

$y=5t$

$z=-1+3t$

f) $P_1(2,5,-1)$, $P_2(12,-5,-7)$

$\vec{P_1P_2}=(10,-10,-6)$

$\vec{d}=(5,-5,-3)$

$\vec{r}=(2,5,-1)+t(5,-5,-3)$

$x=2+5t$

$y=5-5t$

$z=-1-3t$

3a) $(x-4,y-1,z+3)\cdot(2,6,-1)=0$

$2x-8+6y-6-z-3=0$

$2x+6y-z-17=0$

b) $(x+2,y,z-5)\cdot(0,7,0)=0$

$7y=0$

$y=0$

c) $(x-3,y+1,z+2)\cdot(4,-3,0)=0$

$4x-12-3y-3=0$

$4x-3y-15=0$

d) $(x,y,z)\cdot(6,-5,3)=0$

$6x-5y+3z=0$

3e) $(x-4, y-1, z-8)\cdot(11,-6,0)=0$

$11x-44-6y+66=0$

$11x-6y+22=0$

f) $(x-2, y-5, z-1)\cdot(1,1,-1)=0$

$x-2+y-5-z+1=0$

$x+y-z-6=0$

4. $\vec{r}=(2,1,0)+s(1,-1,3)+t(2,0,-5)$

$\begin{matrix} -1 & 0 \\ 3 & -5 \\ 1 & 2 \\ -1 & 0 \end{matrix}$

$\vec{n}=(1,-1,3)\times(2,0,-5)$

$=(5-0, 6+5, 0+2)$

$=(5, 11, 2)$

$(x-2, y-1, z)\cdot(5,11,2)=0$

$5x-10+11y-11+2z=0$

$5x+11y+2z-21=0$

5. L_1: $\vec{r}=(3,0,2)+t(1,-2,2)$

L_2: $\vec{r}=(0,-5,0)+t(-3,2,-10)$

L_3: $\vec{r}=(1,-6,0)+t(4,-1,1)$

Plane: $4x+y-z=10$

$\vec{n}=(4,1,-1)$

Check w/ dot product

$(1,-2,2)\cdot(4,1,-1)=0$ ($\perp$, so L_1 is $\parallel$ to the plane)

$(-3,2,-10)\cdot(4,1,-1)=0$ ($\perp$, so L_2 is $\parallel$ to the plane)

$(4,-1,1)\cdot(4,1,-1)=14$ (L_3 is not $\parallel$ to the plane)

Is $(3,0,2)$ on the plane?

$4(3)+0-2=10$

$10=10$ $\therefore L_1$ is on the plane.

Is $(0,-5,0)$ on the plane?

$4(0)+(-5)-0=10$ $\therefore L_2$ is not on the plane.

$-5\neq 10$

6a) $A(1,0,-1)$, $B(2,0,0)$, $C(6,-1,5)$

$\vec{AC}\times\vec{AB}$

$\begin{matrix} -1 & 0 \\ 6 & 1 \\ 5 & 1 \\ -1 & 0 \end{matrix}$

$\vec{AC}=(5,-1,6)$

$\vec{AB}=(1,0,1)$

$\vec{n}=(-1, 6-5, 1)$

$=(-1,1,1)$

$-x+y+z+D=0$

$-2+D=0$

$D=2$

$-x+y+z+2=0$

$-x-y-z-2=0$

7. $\vec{d_1}=(0,1,0)$

$\vec{d_2}=(1,3,3)$

$\vec{r}=(1,-4,3)+s(0,1,0)+t(1,3,3)$

$\vec{d_1}\times\vec{d_2}$

$\begin{matrix} 1 & 3 \\ 0 & 3 \\ 0 & 1 \\ 1 & 3 \end{matrix}$

$\vec{n}=(3,0,-1)$

$3(1)-1(3)+D=0$

$D=0$

$3x-z=0$

b) $P(4,1,-2)$, $Q(6,4,0)$, $R(0,0,-3)$

$PQ\times PR$

$\begin{matrix} 3 & -1 \\ 2 & -1 \\ 2 & -4 \\ 3 & -1 \end{matrix}$

$\vec{PQ}=(2,3,2)$

$\vec{PR}=(-4,-1,-1)$

$\vec{n}=(-3+2, -8+2, -2+12)$

$=(-1,-6,10)$

$-x-6y+10z+D=0$

$10(-3)+D=0$

$D=30$

$-x-6y+10z+30=0$

9.1 The Intersection of a Line with a Plane and the Intersection of Two Lines

1a) $L_1: x = 1+5s,\ y = 2+s,\ z = -3+s$
$\pi_1: x - 2y - 3z - 6 = 0$

b) $1+5s-4-2s+9-3s-6=0$
$0=0$
∴ The line is on the plane.

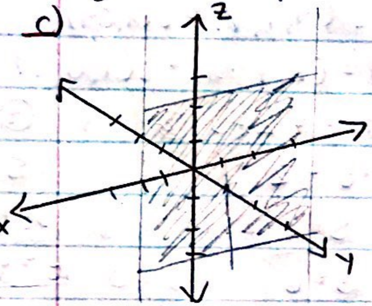
2a) At a single point, never (the line is ∥ to the plane), or always (the line is on the plane)

b) both the line + the plane travel in a single direction

3a) $\vec{r} = s(1,0,0)$ is the x-axis.

b) $y = 1$ is a plane that is parallel to the xz-plane.

c)



d) The line + the plane never intersect.

4a) $L: \vec{r} = (-2,1,2) + t(1,-1,3)$
$\pi: x + 4y + z - 4 = 0$
$\vec{n} = (1,4,1)$
$\vec{n}\cdot\vec{d} = (1,4,1)\cdot(1,-1,3)$
$= 1-4+3$
$= 0$ ∴ The line lies on the plane

$(-2)+4(1)+2-4=0$
$0=0$ ☺

b) $L: \vec{r} = (1,5,6) + t(1,-2,-2)$
$\pi: 2x-3y+4z-11=0$
$\vec{n}\cdot\vec{d} = (2,-3,4)\cdot(1,-2,2)$
$= 2+6-8$
$= 0$
$2(1)-3(5)+4(6)-11=0$
$0=0$ ☺

plane because the direction vector for the line is ⊥ to the normal to the plane and the point is shared.

5a) $\vec{r} = (-1,1,0) + s(-1,2,2)$
$2x-2y+3z-1=0$
$2(-1-s)-2(1+2s)+3(2s)-1=0$
$-2-2s-2-4s+6s-1=0$
$-5\neq 0$
∴ The line does not intersect with the plane

b) $x = 1+2t,\ y = -2+5t,\ z = 1+4t$
$2x-4y+4z-13=0$
$2(1+2t)-4(-2+5t)+4(1+4t)-13=0$
$2+4t+8-20t+4+16t-13=0$
$1\neq 0$
∴ The line + the plane do not intersect.
$2(1)-4(-2)+4(1)-13=0?$
$2+8+4-13=0?$
$1\neq 0$

6a) $(-1,2,2)\cdot(2,-2,3) = \vec{d}\cdot\vec{n}$
$\vec{d}\cdot\vec{n} = -2-4+6$
$= 0$ ∴ $\vec{d}\perp\vec{n}$ but P is not on π.
$2(-1)-2(1)+3(0)-1=0$?
$-2-2-1=0?$
$-5\neq 0$

b) $\vec{d}\cdot\vec{n} = (2,5,4)\cdot(2,-4,4)$
$= 4-20+16$
$= 0$
∴ $\vec{d}\perp\vec{n}$, P is not on π.

Hilary

7a) $\vec{r} = (-1, 3, 4) + p(6, 1, -2)$

$x + 2y - z + 29 = 0$

$(-1+6p) + 2(3+p) - (4-2p) + 29 = 0$

$-1 + 6p + 6 + 2p - 4 + 2p + 29 = 0$

$10p + 30 = 0$

$p = -3$

Point: $\vec{r} = (-1, 3, 4) + -3(6, 1, -2)$

$= (-1, 3, 4) + (-18, -3, 6)$

$= (-19, 0, 10)$

b) $\frac{x-1}{4} = \frac{y+2}{-1} = \frac{z-3}{1}$

$2x + 7y + z + 15 = 0$

$2(1+4t) + 7(-2-t) + (3+t) + 15 = 0$

$2 + 8t - 14 - 7t + 3 + t + 15 = 0$

$2t = -6$

$t = -3$

$\vec{r} = (1, -2, 3) + -3(4, -1, 1)$

$= (1, -2, 3) + (-12, +3, -3)$

$= (-11, 1, 0)$

8a) $\vec{r} = (3, 1, 5) + s(4, -1, 2)$ ①

$x = 4 + 13t,\ y = 1 - 5t,\ z = 5t$ ②

direction vectors are not scalar multiples.

① $4 + 13t = 3 + 4s$ ② $1 - 5t = 1 - s$ ③ $5 + 4s = 5t$

$4 + 13(-\frac{1}{3}) = 3 + 4(-\frac{5}{3})$ $s = 5t$ $5 + 4(5t) = 5t$

$\boxed{-\frac{1}{3} \neq -\frac{11}{3}}$ $\boxed{s = -\frac{5}{3}}$ $5 + 20t = 5t$

$5 = -15t$

These lines are skew! Check in ①. $\boxed{-\frac{1}{3} = t}$

b) $\vec{r} = (3, 7, 2) + m(1, -6, 0)$

$\vec{r} = (-3, 2, 8) + s(7, -1, -6)$

① $3 + m = -3 + 7s$ ② $7 - 6m = 2 - s$ ③ $2 = 8 + 6s$

$3 + 1 = -3 + 7(1)$ $7 - 6m = 2 - 1$ $-6 = -6s$

$4 = 4$ ☺ $-6m = -6$ $1 = s$

$m = 1$

Coordinates:

$\vec{r} = (3, 7, 2) + (1, -6, 0)$

$= (4, 1, 2)$

9a) $\vec{r} = (-2, 3, 4) + p(6, -2, 3)$

$\vec{r} = (-2, 3, -4) + q(6, -2, 11)$

① $-2 + 6p = -2 + 6q$ ② $3 - 2p = 3 - 2q$ ③ $4 + 3p = -4 + 11q$

$6p = 6q$ $-2p = 2q$ $3p = 11q$

$p = q$ $p = q$ $3p = 11p$

∴ These lines are not skew. $p = 0$

9b) $\vec{r} = (4, 1, 6) + t(1, 0, 4)$

$\vec{r} = (2, 1, -8) + s(1, 0, 5)$

① $4 + t = 2 + s$ ② $1 = 1$ ③ $6 + 4t = -8 + 5s$

$s - t = 2$ $5s - 4t = 14$

$s = t + 2$ $5t + 10 - 4t = 14$

$\boxed{s = 6}$ $y = 0$ $\boxed{t = 4}$

These lines are not skew.

c) $\vec{r} = (2, 2, 1) + m(1, 1, 1)$

$\vec{r} = (-2, 2, 1) + p(3, -1, -1)$

① $2 + m = -2 + 3p$ ② $2 + m = 2 - p$

$m - 3p = -4$ $p + m = 0$

$m = 3p - 4$ $p + 3p - 4 = 0$

$m = -1$ $4p = 4$

③ $1 + m = 1 - p$ $p = 1$

$1 + (-1) = 1 - 1$ ∴ The lines are not skew.

$0 = 0$ ☺

d) $\vec{r} = (9, 1, 2) + m(5, 0, 4)$

$\vec{r} = (8, 2, 3) + s(4, 1, -2)$

① $9 + 5m = 8 + 4s$ ② $1 = 2 + s$

$9 + 5m = 8 - 4$ $-1 = s$

$5m = -5$

$m = -1$

③ $2 + 4m = 3 - 2s$ ∴ The lines are skew.

$2 - 4 = 3 + 2$

$-2 \neq 5$

8.3

10. $\vec{r} = (-3,2,1) + s(3,-2,7)$

POI: $(0,0,q)$

$x = -3+3s \quad y = 2-2s \quad z = 1+7s$

$0 = -3+3s \quad 0 = 2-2s \quad q = 1+7s$

$1 = s \quad 1 = s \quad \boxed{q = 8}$

11. $\vec{r} = (-2,3,4) + s(7,-2,2)$

$\vec{r} = (-30,11,-4) + t(7,-2,2)$

a) ① $x = -2+7s$ ② $x = -30+7t$

$y = 3-2s \quad y = 11-2t$

$z = 4+2s \quad z = -4+2t$

① $-2+7s = -30+7t$ ② $3-2s = 11-2t$

$7s-7t = -28 \quad 2s-2t = -8$

$\boxed{s-t=-4} \longleftrightarrow \boxed{s-t=-4}$

③ $4+2s = -4+2t$ all the same

$2s-2t = -8$ ∴ Coincident

$\boxed{s-t=-4}$

b) $-2 = -30+7t \quad 3 = 11-2t \quad 4 = -4+2t$

$28 = 7t \quad -8 = -2t \quad 8 = 2t$

$4 = t \quad 4 = t \quad 4 = t$

∴ The point from L_1 is on L_2, and their direction vectors are the same, so the lines are coincident.

14. $\vec{r} = (2,1,1) + p(4,0,-1)$

$\vec{r} = (3,-1,1) + q(9,-2,-2)$

① $2+4p = 3+9q$ ② $1 = -1-2q$ ③ $1-p = 1-2q$

$2+4(-2) = 3+9(-1) \quad 2 = -2q \quad 2q-p = 0$

$2-8 = 3-9 \quad -1 = q \quad 2(-1)-p = 0$

$-6 = -6 \quad p = -2$

a) $\vec{r} = (2,1,1) + (-2)(4,0,-1)$

$= (2,1,1) + (-8,0,2)$

$= (-6,1,3) \leftarrow$ POI

12. $\vec{r} = (-3,8,1) + s(1,-1,1)$

$\vec{r} = (1,4,2) + t(-3,k,8)$

① $-3+s = 1-3t$ ② $8-s = 4+kt$ ③ $1+s = 2+8t$

$s+3t = 4 \quad 8 - \frac{35}{11} - 4 = \frac{3}{11}k \quad s-8t = 1$

⊖ $s-8t = 1 \quad 88-35-44 = 3k \quad s - \frac{24}{11} = 1$

$11t = 3 \quad 9 = 3k \quad s = \frac{35}{11}$

$t = \frac{3}{11} \quad \boxed{3 = k}$

b) $\vec{r} = (1,4,2) + \frac{3}{11}(-3,3,8)$

$= (1,4,2) + (-\frac{9}{11}, \frac{9}{11}, \frac{24}{11})$

$= (\frac{2}{11}, \frac{53}{11}, \frac{48}{11})$

13. $\vec{r} = (-8,-6,-1) + s(2,2,1)$

$\vec{r} = (0,A,0) + t(1,0,1)$

① $-8+2s = t$ ② $-6+2s = A$ ③ $-1+s = t$

$-8+2s = -1+s \quad -6+14 = A \quad t = 6$

$s = 7 \quad \boxed{8 = A} \quad$ POI: $(6,8,6)$

$\vec{r} = (B,0,0) + p(0,1,1)$

① $-8+2s = B$ ② $-6+2s = p$ ③ $-1+s = p$

$-8+10 = B \quad -6+2s = -1+s \quad 4 = p$

$\boxed{2 = B} \quad s = 5 \quad$ POI $(2,4,4)$

$|\vec{AB}| = |(-4,-4,-2)|$

$= \sqrt{16+16+4}$

$= \sqrt{36}$

$= 6$

15. $\vec{r} = (-1,3,2) + s(5,-2,10)$

$\vec{r} = (4,-1,1) + t(0,2,11)$

① $-1+5s = 4$ ② $3-2s = -1+2t$ ③ $2+10s = 1+11t$

$5s = 5 \quad 3-2 = -1+2 \quad 2+10 = 1+11t$

$s = 1 \quad 1 = 1$ ☺ $\quad 11 = 11t$

$1 = t$

a) POI: $\vec{r} = (-1,3,2) + (5,-2,10)$

$= (4,1,12)$

$\vec{n} = (-42,55,10)$

b) $\vec{r} = (4,1,12) + m(42,55,-10)$

Hilroy

9.2 Systems of Equations

1a) $tx - \frac{t}{t}y = 3$

$\frac{t}{t}y = tx - 3$

$y = t^2x - 3k$

$\therefore$ linear

b) not linear (sinusoidal)

c) not linear (plane)

d) $y = \frac{1}{x} - 3$ (not linear, rational)

2a) $x + y + z = -7$

$x - y + z = -15$

$x - y - z = 1$

b) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & -7 \\ 1 & -1 & 1 & -15 \\ 1 & -1 & -1 & 1 \end{array}\right]$

$R_2 - R_1 \to R_2 \downarrow R_3 - R_1 \to R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -7 \\ 0 & -2 & 0 & -8 \\ 0 & -2 & -2 & 8 \end{array}\right] \leftarrow y = 4$

$-2(4) - 2z = 8$

$-2z = 16$

$z = -8$

$x + 4 - 8 = -7$

$x = -3$

3a) ① $(-7) - 3(5) + 4(\frac{3}{4}) \stackrel{?}{=} -19$

$-7 - 15 + 3 = -19$

$-19 = -19$:)

② $-7 - 8(\frac{3}{4}) = -13$

$-7 - 6 = -13$

$-13 = -13$:)

③ $-7 + 2(5) = 3$

$3 = 3$:)

$\therefore (-7, 5, \frac{3}{4})$ is a solution

b) ① $3(-7) - 2(5) + 16(\frac{3}{4}) \stackrel{?}{=} -19$

$-21 - 10 + 12 \stackrel{?}{=} -19$

$-19 = -19$:)

② $3(-7) - 2(5) = -23$

$-21 - 10 \neq -23$

$-31 \neq -23$

$\therefore (7, 5, \frac{3}{4})$ is not a solution.

4a) ① $x = -2$

② $3y = -9$

$y = -3$

$(-2, -3)$

b) ① $3x + 5y = -21$

$\ominus$ ② $3x - 9y = 21$

$14y = -42$

$y = -3$

$3x + 5(-3) = -21$

$3x = -6$

$x = -2$

$(-2, -3)$

$\therefore$ The systems are equivalent.

5a) ① $2x - y = 11$

$2\times$② $2x + 10y = 22$ $\ominus$

$-11y = -11$

$\boxed{y = 1}$

$2x - 1 = 11$

$2x = 12$

$\boxed{x = 6}$

$\therefore (6, 1)$

b) $3\times$① $6x + 15y = 57$

$2\times$② $\ominus$ $6x + 8y = 22$

$7y = 35$

$\boxed{y = 5}$

$2x + 5(5) = 19$

$2x + 25 = 19$

$2x = -6$

$\therefore (-3, 5)$ $\boxed{x = -3}$

5c) $3\times$① $-3x + 6y = 30$

② $\oplus$ $3x + 5y = 3$

$11y = 33$

$y = 3$

$-x + 2(3) = 10$

$x = -4$

$\therefore (-4, 3)$

6a) $2x + y = 3$

$\ominus$ $2x + y = 4$

$0 = -1$

$\therefore$ The lines do not intersect.

b) $7x - 3y = 9$

② $\ominus$ $7x - 3y = 9$

$0 = 0$

$\therefore$ They are the same line.

7a) Let $y = t$

$x = \frac{3+t}{2}$

b) Let $x = t$, $y = s$

$z = -t + 2s$

9.3 The Intersection of Two Planes

1a) This system means that the planes are parallel.

b) $x - y + z = 1$

$x - y + z = -2$

2a) $z = t$

$y = s$ ∴ $(\frac{1}{2}s - t, s, t)$

$x = \frac{\cancel{1}s - 2t}{2}$ The planes are coincident.

$x = \frac{1}{2}\frac{1}{2}s - t$

b) $2x - y + 2z = 1$ ①

$2x - y + 2z = 1$ ②

3a) $z = -2$

$x - y = 1$ Let $y = t$

$x = t + 1$

∴ $(t+1, t, -2)$

The planes intersect along the line $x = 1 + t, y = t, z = -2$.

b) $x - y + z = -1$

$x - y + 3z = -5$

4a) $m = \frac{1}{2}$, $p = 2q$

not unique!

b) $m = \frac{1}{2}$, $p \neq 2q$

not unique!

c) $(2, 1, 6) \cdot (1, m, 3) = 0$

$2 + m + 18 = 0$

$m = -20$

Yes, this is a unique value.

d) p and q can be any value as long as $m = -20$.

5a) $x + 2y - 3z = 0$

$y = -3s$

$x + 2(-3s) - 3s = 0$

$x = 9s$

$(9s, -3s, s)$

b) $x + 2y - 3z$

$3z = -t$

$z = -\frac{1}{3}t$

$x + 2t - 3(-\frac{1}{3}t) = 0$

$x + 3t = 0$

$x = -3t$

$(-3t, t, -\frac{1}{3}t)$

c) $x - 3$

Same result!

6a) $x + y + z = 1$

$x + y + z = 1$

$0 = 0$

∴ They are coincident planes.

b) $2x - y + z = -1$

⊖ $2x - y + z = -2$

$0 \neq 1$

∴ They do not intersect.

c) $x - y + 2z = 2$

⊖ $\underline{x + y + 2z = 2}$

$-2y = -4$

$y = 2$

∴ They intersect along a line.

d) $x + y + 2z = 4$

⊖ $\underline{x - y \quad = 6}$

$2y + 2z = -2$

∴ They intersect along a line.

7a) Let $z = t$, $y = s$

$x = -s - t$

$(1 - s - t, s, t)$

b) no solution

c) Let $z = t$

$x - 2 + 2t = -2$

$x = -2t + 2t$

$(-2t, -2, t)$

7d) Let $z = t$

$2y = -2t + 2$

$y = -t - 1$

$x + t - 1 + 2(t) = 4$

$x + t = 5$

$x = 5 - t$

$(5 - t, -t - 1, t)$

6e) $2x-y+2z=2$
$-x+2y+z=1$

$$\left[\begin{array}{ccc|c} 2 & -1 & 2 & 2 \\ -1 & 2 & 1 & 1 \end{array}\right] \xrightarrow{2R_2+R_1\to R_2} \left[\begin{array}{ccc|c} 2 & -1 & 2 & 2 \\ 0 & 3 & 4 & 4 \end{array}\right]$$

$\therefore$ They intersect on a line.

$3y+4z=4$ Let $y=s$
$4z=4-3s$
$z=1-\frac{3}{4}s$

7e) $2x-s+2(1-\frac{3}{4}s)=2$
$2x-s+2-\frac{3}{2}s=2$
$2x=\frac{5}{2}s$
$x=\frac{5}{4}s$

$(\frac{5}{4}s, s, 1-\frac{3}{4}s)$

6f) They intersect along a line.

7f) Let $y=s$ $x-s+2(4)=0$
$x=s-8$

$(s-8, s, 4)$

8a) $k=2$
Let $z=t$, $y=s$
$x=1-s-2t$
$(1-s-2t, s, t)$

b) No → The planes need to be parallel and non-coincident, but $k=k$ and the only value for k that creates $\parallel$ normal vectors is 2.

9. ① Find the line of intersection.

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$$ Let $z=t$
$y=-4t$

$2x+4t+t=0$
$x=-\frac{5}{2}t$

$\therefore$ Line of intersection:
$x=-\frac{5}{2}t$
$y=-4t$
$z=t$

② Use the direction vector + the point.
$\vec{r}=(-2,3,6)+t(-\frac{5}{2},4,1)$

10. ① Find the line of intersection:

$$\left[\begin{array}{ccc|c} 2 & -1 & 2 & 0 \\ 2 & 1 & 6 & 4 \end{array}\right] \xrightarrow{R_2-R_1\to R_2} \left[\begin{array}{ccc|c} 2 & -1 & 2 & 0 \\ 0 & 2 & 4 & 4 \end{array}\right]$$

Let $z=t$
$y=2-2t$
$2x-2+2t+2t=0$
$x=1-2t$

$$\boxed{\begin{aligned} x&=1-2t \\ y&=2-2t \\ z&=t \end{aligned}}$$

② Check in the equation.
$5(1-2t)+3(2-2t)+16(t)-11=0$
$5-10t+6-6t+16t-11=0$
$0=0$ $\therefore$ The line is on the plane.

11a) $$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 3 \\ 1 & -2 & 1 & -1 \end{array}\right] \xrightarrow{2R_2-R_1\to R_2} \left[\begin{array}{ccc|c} 2 & 1 & -3 & 3 \\ 0 & -5 & 5 & -5 \end{array}\right]$$

Let $z=t$
$y=1+t$
$2x+1+t-3t=3$
$x=1+t$
$x=1+t,\ y=1+t,\ z=t$

b) $A(a,b,0)$ $B(0,0,c)$
$a=b=1$ $t=0$ $x=1+t$
$A(1,1,0)$ $0=1+t$ $(0,0,-1)$
$-1=t$
$\vec{AB}=(-1,-1,-1)$
$|\vec{AB}|=\sqrt{3}$

Scanned with CamScanner

8a) $y = -2x - 11$

b) $x = 3t + 3$

$\frac{1}{3}x - 1 = t$

$y = -6(\frac{1}{3}x - 1) - 17$

$y = -2x - 11$ ☺

9 $x + y = 6$

$2x + 2y = k$

a) no solution $\Rightarrow k \neq 12$

b) not possible

c) $k = 12$

10a) infinitely many

b) Let $x = t$: $4y = -2t + \frac{11}{4}$

$y = -\frac{1}{2}t + \frac{11}{4}$

c) no integer solutions because of the odd value of the constant.

11a) $2\times$① $2x + 6y = 2a$

② $2x + 3y = b$

$\therefore 3y = 2a - b$

$y = \frac{2}{3}a - \frac{1}{3}b$

$x + 3(\frac{2}{3}a - \frac{1}{3}b) = a$

$x + 2a - b = a$

$x = -a + b$

b) The lines have different slopes, so they will cross

12a) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & -5 \end{array}\right] \xrightarrow{R_1 - R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 1 & -1 & -5 \end{array}\right]$

$\xrightarrow{2R_3 - R_2 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & -3 & -9 \end{array}\right]$

$z = 3$

$2y + 3 = -1$

$2y = -4$

$y = -2$

$x + (-2) + 3 = 0$

$x = -1$

$\therefore (-1, -2, 3)$

12b) $\left[\begin{array}{ccc|c} 2 & -3 & 1 & 6 \\ 1 & 1 & 2 & 31 \\ 1 & -2 & -1 & -17 \end{array}\right] \xrightarrow[R_3 - R_2 \to R_3]{2R_2 - R_1 \to R_2} \left[\begin{array}{ccc|c} 2 & -3 & 1 & 6 \\ 0 & 5 & 3 & 56 \\ 0 & -3 & -3 & -48 \end{array}\right] \xrightarrow{R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 2 & -3 & 1 & 6 \\ 0 & 5 & 3 & 56 \\ 0 & 2 & 0 & 8 \end{array}\right] \leftarrow y = 4$

$2x - 3(4) + 12 = 6$

$2x = 6$

$x = 3$

$5(4) + 3z = 56$

$3z = 36$

$z = 12$

$\therefore (3, 4, 12)$

c) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 10 \\ 0 & 1 & 1 & -2 \\ 1 & 0 & 1 & -4 \end{array}\right] \xrightarrow{R_3 - R_1 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 10 \\ 0 & 1 & 1 & -2 \\ 0 & -1 & 1 & -14 \end{array}\right] \xrightarrow{R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 10 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 2 & -16 \end{array}\right]$

$x + 6 = 10$

$x = 4$

$y - 8 = -2$

$y = 6$

$z = -8$

$\therefore (4, 6, 8)$

d) $\left[\begin{array}{ccc|c} \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & 14 \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{3} & -21 \\ \frac{1}{5} & \frac{1}{3} & \frac{1}{4} & 7 \end{array}\right] \xrightarrow[60R_3 \to R_3]{60R_1 \to R_1,\ 60R_2 \to R_2} \left[\begin{array}{ccc|c} 20 & 15 & 12 & 840 \\ 15 & 12 & 20 & -1260 \\ 12 & 20 & 15 & 420 \end{array}\right] \xrightarrow[3R_1-4R_2 \to R_2]{3R_1-5R_3 \to R_3} \left[\begin{array}{ccc|c} 20 & 15 & 12 & 840 \\ 0 & -3 & -44 & 7560 \\ 0 & -55 & -39 & 420 \end{array}\right]$

$\xrightarrow{55R_2-3R_3 \to R_3} \left[\begin{array}{ccc|c} 20 & 15 & 12 & 840 \\ 0 & -3 & -44 & 7560 \\ 0 & 0 & -2303 & 414540 \end{array}\right]$

$z = -180$

$\frac{1}{3}x + \frac{1}{4}(120) + \frac{1}{5}(-180) = 14$

$\frac{1}{3}x + 30 - 36 = 14$

$x = 60$

$-3y - 44(-180) = 7560$

$y = 120$

$\therefore (60, 120, -180)$

e) $\left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 7 \\ -1 & 0 & 2 & 0 \end{array}\right] \xrightarrow{2R_3+R_1 \to R_3} \left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 7 \\ 0 & -1 & 4 & 0 \end{array}\right] \xrightarrow{2R_3+R_2 \to R_3} \left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 7 \\ 0 & 0 & 7 & 7 \end{array}\right]$

$2x - 4 = 0$

$x = 2$

$2y - 1 = 7$

$y = 4$

$z = 1$

$\therefore$ POI $(2, 4, 1)$

f) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 13 \\ 0 & 2 & -3 & -12 \\ 1 & -1 & 4 & 19 \end{array}\right] \xrightarrow{R_3-R_1 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 13 \\ 0 & 2 & -3 & -12 \\ 0 & -2 & 2 & 6 \end{array}\right] \xrightarrow{R_2+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 13 \\ 0 & 2 & -3 & -12 \\ 0 & 0 & -1 & -6 \end{array}\right]$

$x + 3 + 2(6) = 13$

$x = -2$

$2y - 3(6) = -12$

$2y = 6$

$y = 3$

$z = 6$

$\therefore$ POI $(-2, 3, 6)$

14 $\left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & 1 & 0 & b \\ 0 & 1 & 1 & c \end{array}\right] \xrightarrow{R_2-R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 0 & -1 & b-a \\ 0 & 1 & 1 & c \end{array}\right]$

$\therefore z = a - b$

$y + a - b = c$

$y = c - a + b$

$x + c \cancel{-a} + \cancel{b} + \cancel{a} - \cancel{b} = a$

$x = a - c$

$(a-c, -a+b+c, a-b)$

Scanned with CamScanner

9.4 The Intersection of Three Planes

1. ① $x - 3y + z = 2$
② $y - z = -1$
③ $3z = -12$

a) $z = -4$ $\quad y = -5$ $\quad x - 3(-5) - 4 = 2$ $\quad x = -9$ $\quad (-9, 5, -4)$

b) The planes intersect at a single point.

2. ① $x - y + z = 4$
② $0 = 0$
③ $0 = 0$

a) $x - y + z = 4$
$x - y + z = 4$
$x - y + z = 4$

b) coincident planes.

c) Let $x = s$, $y = t$
$z = 4 - s + t$
$(s, t, 4 - s + t)$

d) $z = t$, $y = s$
$x = 4 + s - t$
$(4 + s - t, s, t)$

3. ① $2x - y + 3z = -2$
② $x - y + 4z = 3$
③ $0 = 1$

a) $2x - y + 3z = -2$
$x - y + 4z = 3$
$2x - y + 3z = -1$

or

$2x - y + 3z = -2$
$x - y + 4z = 3$
$x - y + 4z = 4$

b) Two planes are parallel, so there are no solutions.

4. ① $x + 2y - z = 4$
② $x - 2z = 0$
③ $2x = -6$

a) $x = -3$, $z = -\frac{3}{2}$
$-3 + 2y + \frac{3}{2} = 4$
$2y = \frac{11}{2}$
$y = \frac{11}{4}$
$(-3, \frac{11}{4}, -\frac{3}{2})$

b) The planes intersect at a point.

5a) Equations ② + ③ represent the same plane, and ① is not parallel to them, so it's like finding the intersection of 2 planes (a line).

b) $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ -3 & -3 & 3 & 3 \end{array}\right] \xrightarrow[2R_2 - R_1 \to R_2]{\frac{1}{3}R_3 + R_2 \to R_3} \left[\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Let $z = t$: $\quad 3y - 3t = -3$ $\quad y = -1 + t$ $\quad x + 1 - t + t = 1$ $\quad x = 0$

6. ② and ③ have the same normal vector (are parallel), but different D values, so they never meet ($-1004 \div 5 \neq -201$)

7a) $0 = 0$ will have a solution when it is a single equation (use parameters)

b) A system of equations where $0 = 0$ results may not have a solution if two planes are coincident ($0 = 0$), but the third plane is parallel. ex/ $x + y + z = 2$
$\left.\begin{array}{l} x + y + z = 1 \\ x + y + z = 1 \end{array}\right\}$ coincident

parallel.

8a) $\left[\begin{array}{ccc|c} 2 & 1 & -1 & -3 \\ 1 & -1 & 2 & 0 \\ 3 & 2 & -1 & -5 \end{array}\right] \xrightarrow[3R_2-R_3\to R_3]{2R_2-R_1\to R_2} \left[\begin{array}{ccc|c} 2 & 1 & -1 & -3 \\ 0 & -3 & 5 & 3 \\ 0 & -5 & 7 & 5 \end{array}\right] \xrightarrow{5R_2-3R_3\to R_3} \left[\begin{array}{ccc|c} 2 & 1 & -1 & -3 \\ 0 & -3 & 5 & 3 \\ 0 & 0 & 4 & 0 \end{array}\right]$

$2x-1=-3$ $\quad y=-1 \quad z=0$

$x=-1$ $\quad (-1,-1,0)$

b) $\left[\begin{array}{ccc|c} \frac{1}{3} & -\frac{1}{4} & 1 & \frac{7}{8} \\ 2 & 2 & -3 & -20 \\ 1 & -2 & 3 & 2 \end{array}\right] \xrightarrow[6R_1-R_2\to R_2]{2R_3-R_2\to R_3} \left[\begin{array}{ccc|c} \frac{1}{3} & -\frac{1}{4} & 1 & \frac{7}{8} \\ 0 & -\frac{7}{2} & 9 & \frac{101}{4} \\ 0 & -6 & 9 & 24 \end{array}\right] \xrightarrow{R_3-R_2\to R_3} \left[\begin{array}{ccc|c} \frac{1}{3} & -\frac{1}{4} & 1 & \frac{7}{8} \\ 0 & -\frac{7}{2} & 9 & \frac{101}{4} \\ 0 & -\frac{5}{2} & 0 & -\frac{5}{4} \end{array}\right]$

$\frac{1}{3}x-\frac{1}{4}\left(\frac{1}{2}\right)+3=\frac{7}{8}$ $\quad -\frac{7}{2}\left(\frac{1}{2}\right)+9z=\frac{101}{4}$ $\quad y=\frac{1}{2}$

$\frac{1}{3}x=-2$ $\quad 9z=27$

$x=-6$ $\quad z=3$ $\quad (-6,\frac{1}{2},3)$

c) $\left[\begin{array}{ccc|c} 1 & -1 & 0 & -199 \\ 1 & 0 & 1 & -200 \\ 0 & 1 & -1 & 201 \end{array}\right] \xrightarrow{R_2-R_1\to R_2} \left[\begin{array}{ccc|c} 1 & -1 & 0 & -199 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 201 \end{array}\right] \xrightarrow{R_2-R_3\to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & -199 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 2 & -202 \end{array}\right]$

$y-101=-1$ $\quad z=-101$

$x-100=-199$ $\quad y=100$

$x=-99$

$(-99,100,-101)$

d) $\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 4 \end{array}\right]$ $\quad z=-1-4+2$

$z=-3$

$x=4,\ y=2$ $\quad (4,2,-3)$

10a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & -2 & 2 & 4 \\ 1 & 1 & -1 & -2 \end{array}\right] \xrightarrow[2R_3-R_2\to R_3]{2R_1-R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & -4 & -8 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 4 & -4 & -8 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Planes 1 + 2 are coincident. The 3rd intersects them along the line $x=0$ $\quad y=-2+t$ $\quad z=t$.

b) $\left[\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 4 & -2 & 6 & 0 \\ -2 & 1 & -3 & 0 \end{array}\right] \xrightarrow[R_1+R_3\to R_3]{2R_1-R_2\to R_2} \left[\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore$ All 3 planes are coincident. $x=\frac{s-3t}{2}$, $y=s$, $z=t$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2+B^2}} \qquad d = \frac{|\vec{m} \times \vec{QP}|}{|\vec{m}|}$$

9.5 The Distance from a Point to a Line in R^2 and R^3

1a) $d = \frac{|3(-4)+4(5)-5|}{\sqrt{3^2+4^2}} = \frac{3}{5}$

b) $d = \frac{|5(-4)-12(5)+24|}{\sqrt{5^2+12^2}} = \frac{56}{13}$

c) $d = \frac{|9(-4)-40(5)|}{\sqrt{9^2+40^2}} = \frac{236}{41}$

2a) $P(1,3)$

$d = \frac{|2(1)-1(3)+6|}{\sqrt{2^2+1^2}} = \frac{5}{\sqrt{5}}$

b) $P(-24,0)$

$d = \frac{|7(-24)-24(0)-336|}{\sqrt{7^2+24^2}} = \frac{504}{25}$

3a) $\vec{r} = (-1,2)+s(3,4)$ $\quad \vec{n} = (-4,3)$

$-4(-1)+3(2)+C=0$

$C=-10$

$-4x+3y-10=0$

$d = \frac{|-4(-2)+3(3)-10|}{\sqrt{4^2+3^2}} = \frac{7}{5}$

3b) $\vec{r} = (1,0)+t(5,12)$

$\vec{n} = (-12,5)$

$-12(1)+5(0)+C=0$

$C=12$

$-12x+5y+12=0$

$d = \frac{|-12(-2)+5(3)+12|}{\sqrt{12^2+5^2}} = \frac{51}{13}$

c) $\vec{r} = (1,3)+p(7,-24)$

$\vec{n} = (24,7)$

$24(1)+7(3)+C=0$

$C=-45$

$24x+7y-45=0$

$d = \frac{|24(-2)+7(3)-45|}{\sqrt{24^2+7^2}} = \frac{72}{25}$

4a) Sub in $(0,0)$ for (x_0,y_0) and you are left with $|C|$.

b) $d_1 = \frac{12}{5}$ $\quad d_2 = \frac{12}{5}$

$d = d_1 + d_2 = \frac{24}{5}$

c) $P(0,-3)$

$d = \frac{|3(0)-4(-3)+12|}{5} = \frac{24}{5}$

5a) $\vec{r}_1 = (-2,1)+s(3,4)$ ← Point

$\vec{r}_2 = (1,0)+t(3,4) \leftarrow \vec{n} = (-4,3)$

$-4(1)+3(0)+C=0$

$C=4$

$-4x+3y+4=0$

$d = \frac{|-4(-2)+3(1)+4|}{\sqrt{16+9}} = \frac{15}{5} = 3$

b) $\frac{x-1}{4} = \frac{y}{-3}$ $\quad P(1,0)$

$\frac{x}{4} = \frac{y+1}{-3}$

$-3x = 4y+4$

$-3x-4y-4=0$

$d = \frac{|3(1)+4(0)+4|}{5} = \frac{7}{5}$

5c) $2x-3y+1=0$ $\quad P(1,1)$

$d = \frac{|2(1)-3(1)-3|}{\sqrt{2^2+3^2}} = \frac{4}{\sqrt{13}}$ or $\frac{4\sqrt{13}}{13}$

d) $5x+12y=120$ $\quad P(0,10)$

$d = \frac{|5(0)+12(10)+120|}{\sqrt{5^2+12^2}} = \frac{240}{13}$

6a) $\vec{QP} = (0, -2, 1)$

$\vec{m} = (2, -1, 2)$

$\vec{m} \times \vec{QP} = (-5, -2, -4)$

$$\begin{array}{cc} -1 & -2 \\ 2 & 1 \\ 2 & 0 \\ -1 & -2 \end{array}$$

$$d = \frac{\sqrt{25+4+16}}{\sqrt{4+1+4}} = \frac{3\sqrt{5}}{3} = \sqrt{5}$$

b) $\vec{QP} = (-2, -2, 0)$

$\vec{m} = (-4, 5, 20)$

$\vec{m} \times \vec{QP} = (40, -40, 18)$

$$\begin{array}{cc} 5 & -2 \\ 20 & 0 \\ -4 & -2 \\ 5 & -2 \end{array}$$

$$d = \frac{\sqrt{40^2+40^2+18^2}}{\sqrt{4^2+5^2+20^2}} = \frac{\sqrt{3524}}{\sqrt{441}} = \frac{2\sqrt{881}}{21}$$

c) $\vec{QP} = (2, 3, 1)$

$\vec{m} = (12, -3, 4)$

$\vec{m} \times \vec{QP} = (-15, -4, 42)$

$$\begin{array}{cc} -3 & 3 \\ 4 & 1 \\ 12 & 2 \\ -3 & 3 \end{array}$$

$$d = \frac{\sqrt{15^2+4^2+42^2}}{\sqrt{12^2+3^2+4^2}} = \frac{\sqrt{2005}}{13} \approx 3.44$$

9.6 The Distance from a Point to a Plane

1. $d = \frac{|2(-3)+2+2(1)+2|}{\sqrt{2^2+1^2+2^2}}$

a) Yes they have.

b) The point is on the line.

2a) $d = \frac{|20(3)-4(1)+7|}{\sqrt{20^2+4^2+5^2}}$

$= \frac{63}{21}$

$= 3$

2b) $d = \frac{|2(0)+1(-1)+2(0)-8|}{\sqrt{2^2+1^2+2^2}}$

$= \frac{9}{3}$

$= 3$

c) $d = \frac{|3(5)-4(1)+0(4)-1|}{\sqrt{3^2+4^2}}$

$= \frac{10}{5}$

$= 2$

d) $d = \frac{|5(1)-12(0)+0(0)+0|}{\sqrt{5^2+12^2}}$

$= \frac{5}{13}$

e) $d = \frac{|18(-1)-9(0)+18(1)-11|}{\sqrt{18^2+9^2+18^2}}$

$= \frac{11}{27}$

3a) Point of π_1: $(0, 0, -\frac{13}{6})$

$d = \frac{|3(0)+4(0)-12(-\frac{13}{6})+39|}{\sqrt{3^2+4^2+12^2}}$

$= \frac{65}{13}$

$= 5$

b) $\frac{-26+39}{2} = \frac{13}{2}$

$3x+4y-12z+\frac{13}{2}=0$

OR $6x+8y-24z+13=0$

c)

4a) $d = \frac{|0(1)+1+0(-3)+3|}{1}$

$= 4$

b) $d = \frac{|1(-1)-3|}{1}$

$= 4$

c) $d = \frac{|1(1)+1|}{1}$

$= 2$

5. $\vec{AB} = (-4,-3,-1)$

$\vec{AC} = (12, 2, -4)$

$\vec{AB} \times \vec{AC} = (12+2, -12-16, -8+36)$

$\begin{matrix} -3 & 2 \\ -1 & -4 \\ -4 & 12 \\ -3 & 2 \end{matrix}$

$= (14, -28, 28)$

$14(1)-28(2)+28(3)+D=0$

$14-56+84+D=0$

$D=-42$

$14x-28y+28z-42=0$

$7x-14y+14z-21=0$

$d = \frac{|7(1)-14(-1)+14(1)-21|}{\sqrt{7^2+14^2+14^2}}$

$= \frac{14}{21}$ or $\frac{2}{3}$

Nithany

6. $\frac{|A(3)-2(-3)+6(1)|}{\sqrt{A^2+2^2+6^2}} = 3$

$(|3A+12|)^2 = \left(3\sqrt{40+A^2}\right)^2$

$3A^2+72A+144 = 9(40+A^2)$

$3A^2+72A+144 = 360+9A^2$

$6A^2-72A+216=0$

$A^2-12A+36=0$

$(A-6)^2=0$

$\boxed{A=6}$

$6x-2y+6z=0$

$3x-y+3z=0$

$A=3$.

Scanned with CamScanner