Chapter 8: Equations of Lines and Planes
Review of Prerequisite Skills

1a) $(3,-2,1)-(1,7,-5)$
$=(2,-9,6)$

b) $5(2,-3,-4)+3(1,1,-7)$
$=(10,-15,-20)+(3,3,-21)$
$=(13,-12,-41)$

2a) $a(1,-3)+b(4,2)=(-8,-18)$
$a+4b=-8 \quad -3a+2b=-18$
$a=-4b-8 \quad -3(-4b-8)+2b=-18$
$a=-4(-3)-8 \quad 12b+24+2b=-18$
$=4 \quad 14b=-42$
$\therefore 4(1,-3)-3(4,2)=(-8,-18), \quad b=-3$
so the points are collinear.

b) $a(-4,3)+b(4,5)=(0,4)$
$-4a+4b=0 \quad 3a+5b=4$
$b=a \quad 8b=4$
$\therefore \frac{1}{2}(-4,3)+\frac{1}{2}(4,5)=(0,4) \quad b=\frac{1}{2}$
so the points are collinear.

c) $a(1,2,1)+b(4,7,0)=(7,12,-1)$
$a+4b=7 \quad 2a+7b=12 \quad \boxed{a=-1}$
$4b=8$, Check:
$b=2 \quad 2(-1)+7(2)=12$
$12=12$
$\therefore$ The points are collinear.

d) $a(1,2,-3)+b(4,1,3)=(2,4,0)$
$a+4b=2 \quad 2a+b=4 \quad -3a+3b=0$
$\frac{4}{3}+4\left(\frac{4}{3}\right)\neq 2 \quad 3a=4 \quad a=b$
$\frac{4}{3}+\frac{16}{3}\neq 2 \quad a=\frac{4}{3}$
$\frac{20}{3}\neq 2$ $\therefore$ The points are not collinear.

3. $\vec{AB}=(1,-1,5)$
$\vec{BC}=(3,-2,-1)$
$\vec{CA}=(-4,3,-4)$
$\vec{AB}\cdot\vec{BC}=3+2-5$
$=0$
$\therefore \vec{AB}\perp\vec{BC}$, so it is a right Δ.

4. $\vec{u}\cdot\vec{v}=0$
$(t,-1,3)\cdot(2,t,-6)=0$
$2t-t-18=0$
$\boxed{t=18}$

5a) $\vec{a}=(1,-3)$ $\perp$ to $(3,1)$ b) $\vec{b}=(6,-5)$ $\perp$ to $(5,6)$
c) $(-7,4,0)\perp$ to $(-4,7,0)$

$\vec{a}$	$\vec{b}$
10	1
9	-2
4	3
10	1

6. $|\vec{a}\times\vec{b}|=|[(-20)-9,(27)-(-8),(4)-(30)]|$
$=|(-29,35,-26)|$
$=52.36$ sq. units

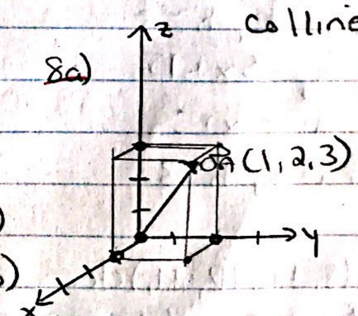
$\vec{a}$	$\vec{b}$
1	-5
-4	-2
2	3
1	-5

7a) $\vec{a}\times\vec{b}$
$=(-2-20,-12+4,-10-3)$
$=(-22,-8,-13)$
Check: $(2,1,-4)\cdot(-22,-8,-13)$
$=-44-8+52$
$=0$ ☺

$\vec{a}$	$\vec{b}$
-2	-1
0	0
-1	-2
-2	-1

b) $\vec{a}\times\vec{b}$
$=(0,0,1-4)$
$=(0,0,-3)$

8a)



9a) $\vec{AB} = (-3-4, 5-8)$
$= (-7, -3)$

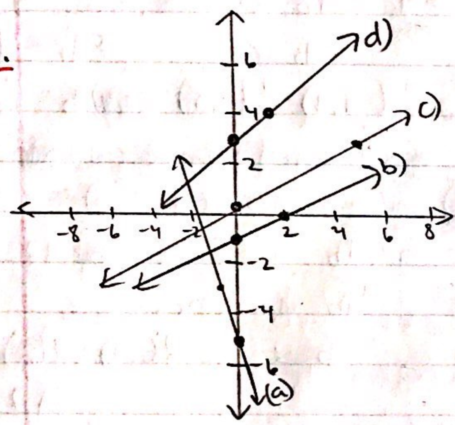
b) $\vec{AB} = (3+7, 8+6)$
$= (10, 14)$

c) $\vec{AB} = (3-1, -6-2, 9-4)$
$= (2, -8, 5)$

d) $\vec{AB} = (0-4, 5-0, 0+4)$
$= (-4, 5, 4)$

10a) $(7, 3)$ b) $(-10, -14)$
c) $(-2, 8, -5)$ d) $(4, -5, -4)$

11.



a) $m = -2$, $b = -5$

b) $-8y = -4x + 8$
$y = \frac{1}{2}x - 1$
$m = \frac{1}{2}$, $b = -1$

c) $-5y = -3x - 1$
$y = \frac{3}{5}x + \frac{1}{5}$
$m = \frac{3}{5}$, $b = \frac{1}{5}$

d) $5y = 5x + 15$
$y = x + 3$
$m = 1$, $b = 3$

12a) $(8, 14)$, $(12, 21)$, etc.
b) $(-10, 8, 6)$, $(-15, 12, 9)$, etc.
c) $\vec{i} + 3\vec{j} - 2\vec{k}$
d) $-10\vec{i} + 16\vec{j} + 4\vec{k}$

any (+) multiple.

13a) $\vec{u} \cdot \vec{v}$
$= (4, -9, -1) \cdot (4, -2, 1)$
$= 16 + 18 - 1$
$= 33$

b) $-\vec{v} \cdot \vec{u}$
$= (-4, 2, -1) \cdot (4, -9, -1)$
$= -16 - 18 + 1$
$= -33$

c) $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v})$
$= (8, -11, 0) \cdot (0, -7, -2)$
$= 0 + 77 + 0$
$= 77$

$\begin{array}{cc} \vec{u} & \vec{v} \\ -9 & -2 \\ -1 & +1 \\ 4 & 4 \\ -9 & -2 \end{array}$

d) $\vec{u} \times \vec{v}$
$= (-9-2, -4-4, -8+36)$
$= (-11, -8, 28)$

$\begin{array}{cc} \vec{v} & \vec{u} \\ -2 & -9 \\ 1 & -1 \\ 4 & 4 \\ -2 & -9 \end{array}$

e) $\vec{v} \times \vec{u}$
$= (2+9, 4+4, -36+8)$
$= (11, 8, -28)$

$\begin{array}{cc} -20 & -5 \\ -1 & -3 \\ 12 & -4 \\ -20 & -5 \end{array}$

f) $(2\vec{u} + \vec{v}) \times (\vec{u} - 2\vec{v})$
$= (12, -20, -1) \times (-4, -5, -3)$
$= (60 - 5, 4 + 36, -60 - 80)$
$= (55, 40, -140)$

14. Dot product → scalar
Cross product → vector ⊥ to the original vectors

8.1 Vector and Parametric Equations of a Line in R^2

1. If $s=-6$, $\vec{m}=(-2,-1)$
If $s=6$, $\vec{m}=(2,1)$
If $s=\frac{6}{7}$, $\vec{m}=(\frac{2}{7},\frac{1}{7})$

2a) Let $t=1 \rightarrow (4,3)$
$t=2 \rightarrow (7,1)$
$t=3 \rightarrow (10,-1)$

b) $1+3t=-14$ $\quad$ $5-2t=15$
$3t=-15$ $\quad$ $-2t=10$
$t=-5$ $\quad$ ☺ $\quad$ $t=-5$

3a) $\vec{m}=(2,1)$; $(3,4)$
b) $\vec{m}=(2,-7)$; $(1,3)$
c) $\vec{m}=(0,2)$; $(4,1)$
d) $\vec{m}=(-5,0)$; $(0,6)$

4. $\vec{m}=(-5,4)$
$(x,y)=(2,1)+s(-5,4)$
or $(x,y)=(-3,5)+t(-5,4)$

5a) $-2-t=-9$ $\quad$ $4+2t=18$
$t=7$ $\quad$ ☺ $\quad$ $2t=14$
$t=7$
$\therefore (-9,18)$ is on the line.
b) $\vec{r}=(-2,4)+t(-1,2)$
c) $\vec{r}=(-9,18)+s(-1,2)$

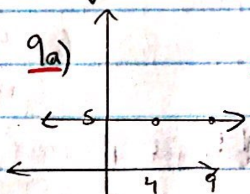
6a) $(0,0),(3,4),(9,12)$
b) $\vec{r}=t(1,1)$
c) same line; $(9,12)$ is on the original line.

7. Changing only the point and not the direction vector changes the direction.

8a)

b) $\vec{r}=t(0,1)$, $x=0$, $y=t$
c)

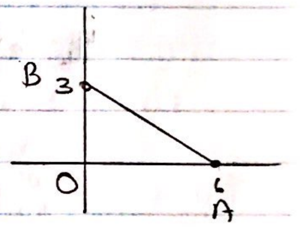
9a)



b) $\vec{r}=(4,5)+t(5,0)$
$x=4+5t$ $\quad$ $y=5$

10a) $L_1: \vec{r}=(2,0)+t(-5,3)$
b) $x=2-5t$
$0=2-5t$ $\quad$ Point: $(0,1\frac{1}{5})$
$-2=-5t$
$\frac{2}{5}=t$

11. $x=-10-2s$, $y=8+s$
$a=-10-2s$ $\quad$ $0=8+s$
$a=-10+16$ $\quad$ $s=-8$
$a=6$



$(6,0)$ $\quad$ $(0,3)$
$0=-10-2s$ $\quad$ $b=8+s$ $\quad$ $A=\frac{6\times 3}{2}$
$-5=s$ $\quad$ $b=8-5$ $\quad$ $=9$ sq-units
$b=3$

12. $A(1,2)$ $B(-1,5)$ $C(-3,8)$ $D(-5,11)$
a) $\vec{AC}=(-4,6)$
$2(\vec{AB})=2(-2,3)$
$=(-4,6)$
$\therefore \vec{AC}=2\vec{AB}$
b) $\vec{AD}=(-6,9)$
$3\vec{AB}=(-6,9)$ ☺
c) $\frac{2}{3}\vec{AD}=(-4,6)$ ☺

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

8.2 Cartesian Equation of a Line

1a) $\vec{m} = (6,-5)$

b) $\vec{m} = (5,6)$

c) $(0,9)$

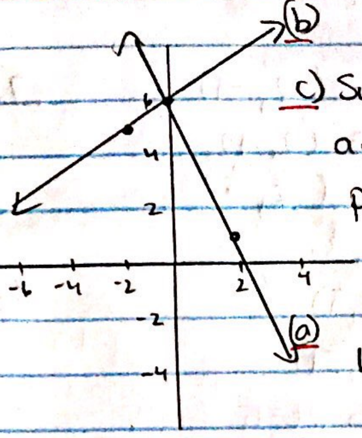
d) $\vec{r} = (7,9) + t(6,-5)$

$x = 7 + 6t \quad y = 9 - 5t$

e) $\vec{r} = (-2,1) + t(5,6)$

$x = -2 + 5t \quad y = 1 + 6t$

2.



c) Switching the point and the vector produces a different line.

3a) $y = \frac{7}{8}x - 6$

$\vec{r} = (0,-6) + t(8,7)$

$x = 8t, \; y = -6 + 7t$

b) $\vec{r} = (0,5) + t(2,3)$

$x = 2t \quad y = 5 + 3t$

c) $\vec{r} = (0,-1) + t(1,0)$

$x = t, \; y = -1$

d) $\vec{r} = (4,0) + t(0,1)$

$x = 4, \; y = t$

4. $\frac{6x - 18y + 24 = 0}{6 \qquad\qquad 6}$

$x - 3y + 8 = 0$

$\therefore$ The lines are the same!

5a) $2x - 3y + 6 = 0$

$y = \frac{2}{3}x + 2$

$4x - 6y + k = 0$

$y = \frac{2}{3}x + \frac{1}{6}k$

The normals are collinear.

b) $k = 12$

6. $4x + 5y + C = 0$

$4(-1) + 5(5) + C = 0$

$21 + C = 0$

$C = -21$

$4x + 5y - 21 = 0$

7. $\vec{AB} = (1,-1)$

$\vec{n} = (1,1)$

$x + y + C = 0$

$-3 + 5 + C = 0$

$C = -2$

$x + y - 2 = 0$

8. $-4y = -2x - 7$

$y = \frac{1}{2}x - 7$

$\vec{n} = (2,1)$

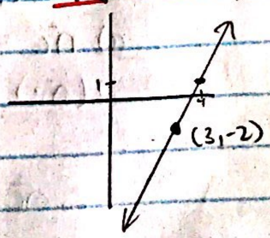
$2x + y + C = 0$

$2(7) + 2 + C = 0$

$C = -16$

$2x + y - 16 = 0$

9a) $\vec{r} = (3,-2) + t(-1,-4)$



b) $4x - y + C = 0$

$4(3) - (-2) + C = 0$

$C = -14$

$4x - y - 14 = 0$

10a) $\cos\theta = \frac{-8+5}{(\sqrt{29})(\sqrt{17})}$

$\theta = 98°$

Acute angle $= 82°$

b) $\cos\theta = \frac{-5-24}{(\sqrt{41})(\sqrt{37})}$

$\theta = 138°$

Acute angle $= 42°$

c) $\vec{n}_1 : (2,1); \; \vec{m}_1 = (-1,2)$

$\vec{n}_2 = (4,-3) \; \vec{m}_2 = (3,4)$

$\cos\theta = \frac{-3+8}{(\sqrt{5})(5)}$

$= \frac{1}{\sqrt{5}}$

$\theta = 63°$

d) $\cos\theta = \frac{-2+8}{(\sqrt{20})(\sqrt{5})}$ $\quad y = \frac{1}{2}x - 2 \quad \vec{n} = (2,1) \quad \vec{m} = (-1,2)$

$\cos\theta = \frac{3}{5}$

$\theta = 53°$

e) $\cos\theta = \frac{-8+(-5)}{(\sqrt{29})(\sqrt{17})}$

$= 125.8°$

Acute angle $= 54°$

f) $y = \frac{1}{2}x + 2 \quad \vec{n} = (2,1) \quad \vec{m} = (-1,2)$

$x = 3$

$\vec{r} = (3,0) + t(0,1)$

$\cos\theta = \frac{0-2}{(1)(\sqrt{5})}$

$\theta = 153°$

Acute angle: $27°$

11. $L_1: -3y = -x + 6 \quad \vec{n}_1 = (3,1)$

$y = \frac{1}{3}x - 2$

$L_2: 2y = -x + 7 \quad \vec{n}_2 = (-2,1)$

$y = -\frac{1}{2}x + \frac{7}{2}$

$\cos\theta = \dfrac{-6+1}{(\sqrt{10})(\sqrt{5})}$

$= \dfrac{-5}{5\sqrt{2}}$

$= -\frac{1}{\sqrt{2}}$

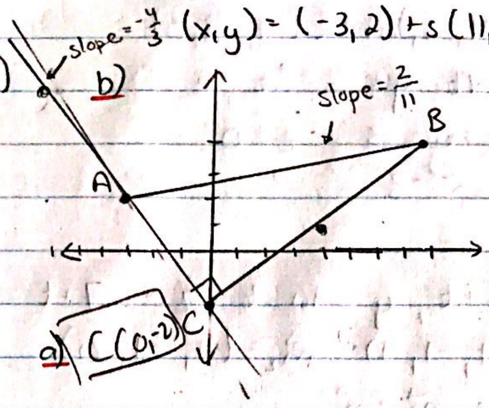
$\theta = 135°$

Acute angle $= 45°$.

12. $\vec{AB} = (11,2)$

$(x,y) = (-3,2) + s(11,2)$

b)



a) $\boxed{C(0,-2)}$

c) $\vec{AC} \cdot \vec{BC}$

$= (3,-4)\cdot(-8,-6)$

$= -24 + 24$

$= 0$

$\therefore \angle ACB = 90°$

8.3 Vector and Parametric and Symmetric Equations of a Line in R^3

For all solutions

1a) $(-3,1,8)$
b) $(1,-1,3)$
c) $(-2,1,3)$
d) $(-2,-3,1)$
e) $(3,-2,-1)$
f) $(\frac{1}{3}, -\frac{3}{4}, \frac{2}{5})$

2a) $(-1,1,9)$
b) $(2,1,-1)$
c) $(3,-4,-1)$
d) $(-1,0,2)$
e) $(0,0,2)$
f) $(2,-1,2)$

3a) $\vec{m} = (4,-5,1)$

① $\vec{r}_1 = (-1,2,4) + t(4,-5,1)$

② $\vec{r}_2 = (3,-3,5) + s(4,-5,1)$

b) $x_1 = -1 + 4t,\ y_1 = 2 - 5t,\ z_1 = 4 + t$

$x_2 = 3 + 4s,\ y_2 = -3 - 5s,\ z_2 = 5 + s$.

4. $\vec{m} = (3,0,0)$

a) $\vec{r} = (-1,5,-4) + t(3,0,0)$

b) $x = -1 + 3t,\ y = 5,\ z = -4$

c) Two coordinates are zero, so you'd only have one part of a symmetric equation. You need at least two!

c) $\vec{m} = (0,6,6) = (0,1,1)$

$\vec{r} = (-2,3,0) + t(0,1,1)$

$x = -2,\ y = 3 + t,\ z = t$

$\dfrac{y-3}{1} = \dfrac{z}{1},\ x = -2$

5a) $\vec{r} = (-1,2,1) + t(3,-2,1)$

$x = -1 + 3t,\ y = 2 - 2t,\ z = 1 + t$

$\dfrac{x+1}{3} = \dfrac{y-2}{-2} = \dfrac{z-1}{1}$

b) $\vec{m} = (0,1,1)$

$\vec{r} = (-1,1,0) + t(0,1,1)$

$x = -1,\ y = 1 + t,\ z = t$

$\dfrac{y-1}{1} = \dfrac{z}{1},\ x = -1$

d) $\vec{m} = (0,1,0)$

$\vec{r} = (-1,0,0) + t(0,1,0)$

$x = -1,\ y = t,\ z = 0$

e) $\vec{m} = (4, -3, 0)$
$\vec{r} = (-4, 3, 0) + t(4, -3, 0)$
$x = -4 + 4t,\ y = 3 - 3t,\ z = 0$
$\frac{x+4}{4} = \frac{y-3}{-3},\ z = 0$

f) $\vec{m} = (0, 0, 1)$
$\vec{r} = (1, 2, 4) + t(0, 0, 1)$
$x = 1,\ y = 2,\ z = 4 + t$
no symmetric equations.

6a) $x = -6 + t,\ y = 10 - t,\ z = 7 + t$; $x = -7 + s,\ y = 11 - s,\ z = 5$
b) $\vec{m}_1 = (1, -1, 1)$ $\vec{m}_2 = (1, -1, 0)$
$\cos\theta = \frac{\vec{m}_1 \cdot \vec{m}_2}{|\vec{m}_1||\vec{m}_2|}$
$= \frac{2}{(\sqrt{3})(\sqrt{2})}$
$= \frac{2\sqrt{6}}{6}$
$= \sqrt{6}/3$
$\theta = 35°$

7. $L_1: \vec{r}_1 = (-7, -1, 5) + t(8, 2, -2)$
$L_2: \vec{r}_2 = (1, 1, 3) + s(-4, -1, 1)$
If $s = -2$, $\vec{m} = (8, 2, -2)$
so the lines have the same direction vector.
Check $(1, 1, 3)$ in line ①:
$\frac{1+7}{8} = \frac{1+1}{2} = \frac{3-5}{-2} = 1$, so $(1, 1, 3)$ is on both lines. They are the same line,

8a) $\frac{x}{-3} = \frac{y}{1} = \frac{z-3}{-6}$
Check $(6, -2, 15)$
$\frac{6}{-3} = \frac{-2}{1} = \frac{15-3}{-6}$
$-2 = -2 = -2$ ☺
Check $(-15, 5, -27)$
$\frac{-15}{-3} = \frac{5}{1} = \frac{-27-3}{-6}$
$5 = 5 = 5$ ☺
∴ The points are on the line.
b) $\vec{AB} = (-21, 7, -42)$
$t(-3, 1, -6)$, $\boxed{t = 7}$
(same direction)
$x = -3t,\ y = t,\ z = -6t + 3$
$-2 \le t \le 5$

9. $L_1: \frac{x-1}{k} = \frac{y-2}{2} = \frac{z+1}{k-1}$ $L_2: \frac{x+3}{-2} = \frac{z}{1}, y = -1$
$\vec{m}_1 = (k, 2, k-1)$ $\vec{m}_2 = (-2, 0, 1)$
$\vec{m}_1 \cdot \vec{m}_2 = 0$
$-2k + k - 1 = 0$
$-k - 1 = 0$
$k = -1$

10a) Let $t = 0$: $(4, -2, 5)$
$t = 1$: $(0, -8, 13)$
$t = -1$: $(8, 4, -3)$
b) Let $s = 0$: $(-4, 2, 9)$
$s = -1$: $(-9, 3, 15)$
$s = 1$: $(1, 1, 3)$
c) $(-1, 2, 0), (2, 1, 4), (5, 0, 8)$
d) $(2, 3, -4), (5, 8, -4), (-1, -2, -4)$

11a) $x = 4 - 4t,\ y = -2 - 6t,\ z = 5 + 8t$
$\frac{x-4}{-4} = \frac{y+2}{-6} = \frac{z-5}{8}$
b) $\vec{r} = (-4, 2, 9) + s(5, -1, -6)$
$\frac{x+4}{5} = \frac{y-2}{-1} = \frac{z-9}{-6}$
c) $x = -1 + 3t,\ y = 2 - t,\ z = 4t$
$\vec{r} = (-1, 2, 0) + t(3, -1, 4)$
d) $x = -4,\ y = 2 + 3t,\ z = 3 + 5t$
$\vec{r} = (2, 3, -4) + t(0, 3, 5)$

8.4 Vector and Parametric Equations of a Plane

1a) plane b) line c) line d) plane

2a) $(4, -24, 9)$

b) $(1, -2, 5)$

c) $\vec{r} = (2, 1, 3) + s(4, -24, 9) + t(1, -2, 5)$

3a) $(0, 0, -1)$

b) $(2, -3, -3)$ and $(0, 5, -2)$

c) $x = 2(-1)$ $\quad y = -3(-1) + 5(-4)$ $\quad z = -1 - 3(-1) + 2(-4)$

$= -2$ $\quad = -17$ $\quad = 10$

$P(-2, -17, 10)$

d) $2m = 0$ $\quad -3m + 5n = 15$ $\quad -1 - 3m - 2n = -7$

$m = 0$ $\quad n = 3$ $\quad -1 - 2(3) = -7$

$\therefore m = 0, n = 3$ $\quad -7 = -7$

e) The equation for z would be untrue if the z-coordinate were -8, so $(0, 15, -8)$ couldn't be in the plane.

4a) $\vec{PQ} = (0, 0, 1)$ $\quad \vec{PR} = (3, -3, 0)$

$\vec{r}_1 = (-2, 3, 1) + s(0, 0, 1) + t(1, -1, 0)$

b) $\vec{QR} = (3, -3, -1)$

$\vec{r}_2 = (-2, 3, 2) + s(0, 0, 1) + t(3, -3, -1)$

5. The direction vectors are the same if $s = 2$, so the vectors are collinear. This is an equation for a line.

6a) $\vec{r} = (-1, 2, 7) + s(4, 1, 0) + t(3, 4, -1)$

$x = -1 + 4s + 3t, \; y = 2 + s + 4t, \; z = 7 - t$

b) $\vec{a} = (-1, 1, 0)$ $\quad \vec{b} = (-1, 0, 1)$

$\vec{r} = (1, 0, 0) + s(-1, 1, 0) + t(-1, 0, 1)$

$x = 1 - s - t, \; y = s, \; z = t$

6c) $\vec{AB} = (3, 4, -6)$

$\vec{r} = (1, 1, 0) + s(7, 1, 2) + t(3, 4, -6)$

$x = 1 + 7s + 3t, \; y = 1 + s + 4t, \; z = 2s - 6t$

7a) $x = 2 + 4s - t$ $\quad y = 2s + t$ $\quad z = 1 - s + 2t$

$5 = 2 + 4s - t$ $\quad$ ③ $3 = 2s + t$ $\quad 2 = 1 - s + 2t$

② $4s - t = 3$ $\quad$ Do those values work in ③? $\quad -s + 2t = 1$

$4(2t - 1) - t = 3$ $\quad 3 = 2(1) + (1)$ $\quad$ ① $s = 2t - 1$

$8t - 4 - t = 3$ $\quad 3 = 3$ $\quad s = 2(1) - 1$

$7t = 7$ $\quad s = 1$

$t = 1$

b) $2 + 4s - t = 0$ $\quad 2s + t = 5$ $\quad 1 - s + 2t = -4$

② $4s - t = -2$ $\quad$ Check: $\quad$ ① $2t + 5 = s$

$4(2t + 5) - t = -2$ $\quad 2\left(-\frac{9}{7}\right) + \left(-\frac{22}{7}\right) = 5$ $\quad 2\left(-\frac{22}{7}\right) + 5 = s$

$8t + 20 - t = -2$ $\quad -\frac{18}{7} - \frac{22}{7} = 5$ $\quad -\frac{44}{7} + 5 = s$

$7t = -22$ $\quad -\frac{40}{7} = 5$ $\quad -\frac{9}{7} = s$

$t = -\frac{22}{7}$ $\quad$ Not true, so not on π.

8a) $l_1: \vec{r}_1 = (-3, 5, 6) + s(-1, 1, 2)$

$l_2: \vec{r}_2 = (-3, 5, 6) + v(2, 1, -3)$

b) POI: $(-3, 5, 6)$

9. $x = 4 + 11s - 7t$ $\quad y = 1 - s + 2t$

① $11s - 7t = -4$ $\quad$ ② $s - 2t = 1$

$z = 6 + 3s - 2t$ $\quad (0, 0, c)$

$c - 6 = 3s - 2t$ $\quad c - 6 = -3 + 2$ $\quad \boxed{c = 5}$

From ②: $s = 1 + 2t$

Into ①: $11(1 + 2t) - 7t = -4$

$11 + 22t - 7t = -4$

$15t = -15$

$\therefore$ It intersects at $(0, 0, 5)$. $\quad \boxed{t = -1}$

$s = 1 - 2$

$\boxed{s = -1}$

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10. $\vec{a} = (-3, 1, -2)$

$\vec{r} = (2, 1, 3) + s(4, 1, 5) + t(-3, 1, -2)$

11. $\vec{a} = (2, -2, -3)$

$\vec{r} = (-2, 2, 3) + m(2, -1, 7) + n(2, -2, -3)$

12a) $(1, 0, 0)$ and $(0, 1, 0)$

$(1, 1, 0)$ and $(-1, 1, 0)$

b) $\vec{r} = (0, 0, 0) + s(1, 0, 0) + t(0, 1, 0)$

$x = s,\ y = t,\ z = 0$

13a) $\vec{OA} = (-1, 2, 5)$, $\vec{OC} = (3, -1, 7)$

$\vec{r} = (-1, 2, 5) + s(-1, 2, 5) + t(3, -1, 7)$

b) $\vec{PQ} = (-1, 2, 5)$ $\quad \vec{PR} = (3, -1, 7)$

$\vec{r} = (-2, 2, 3) + s(-1, 2, 5) + t(3, -1, 7)$

c) Parallel (same direction vectors)

14. From ②:

$x = -s - t,\ y = 5s - 5t,\ z = -3s + 7t$

Use $(-3, 2, 4)$:

$$-3 = -s - t \qquad 2 = 5s - 5t \qquad 4 = -3s + 7t$$
$$s = 3 - t \qquad 2 = 5(3-t) - 5t \qquad 4 = -3\left(\tfrac{17}{10}\right) + 7\left(\tfrac{13}{10}\right)$$
$$s = \tfrac{17}{10} \qquad 2 = 15 - 5t - 5t \qquad 4 = 4 \; ☺$$
$$-13 = -10t$$
$$\tfrac{13}{10} = t$$

$\therefore$ $(-3, 2, 4)$ can be expressed as a linear comb. of the direction vectors of $\vec{r}$.

Use $(-4, 7, 1)$:

$$-4 = -s - t \qquad 7 = 5s - 5t \qquad 1 = -3s + 7t$$
$$s = 4 - t \qquad 7 = 5(4-t) - 5t \qquad 1 = -3\left(\tfrac{27}{10}\right) + 7\left(\tfrac{13}{10}\right)$$
$$s = \tfrac{27}{10} \qquad 7 = 20 - 10t \qquad 1 = 1 \; ☺$$
$$\tfrac{13}{10} = t$$

$\therefore$ $(-4, 7, 1)$ can also be expressed as a linear comb. of the direction vectors of $\vec{r}$.

$\therefore$ Both are in the plane, so the two equations represent the same plane.

15. y-axis $(0, b, 0)$

$$1 + m + n = 0 \qquad 2 + 2m - n = b \qquad 3 + 5m + 3n = 0$$
$$m = -n - 1 \qquad 2 + 2(0) + 1 = b \qquad 3 + 5(-n-1) + 3n = 0$$
$$m = 0 \qquad 3 = b \qquad -2n - 2 = 0$$
$$n = -1$$

$\therefore A(0, 3, 0)$

z-axis $(0, 0, c)$

$$m = -n - 1 \qquad 2 + 2m - n = 0 \qquad 3 + 5m + 3n = c$$
$$m = -1 \qquad 2(-n-1) - n = -2 \qquad 3 - 5 = c$$
$$-3n = 0 \qquad -2 = c$$
$$n = 0$$

$B(0, 0, -2)$

$\vec{AB} = (0, -3, -2)$

$\vec{r} = (0, 3, 0) + t(0, -3, -2)$

8.5 The Cartesian Equation of a Plane

1a) $\vec{n} = (1, -7, -18)$
b) $D = 0$
c) $(0,0,0)$
Let $y=1, z=1$
$(25,1,1)$
Let $y=-1, z=1$
$(-11,-1,1)$

2a) $\vec{n} = (2,-5,0)$
b) $D=0$
c) $(0,0)$
$(5,2)$
$(10,4)$

3a) $\vec{n} = (1,0,0)$
b) $D=0$
c) $(0,1,0)$
$(0,0,1)$
$(0,0,0)$

4a) $x+5y-7z=0$
b) $-8x+12y+7z=0$

5. $Ax+By+Cz+D=0$
$x+7y+5z+D=0$
$(-3)+7(3)+5(5)+D=0$
$-3+21+25+D=0$
$D=-43$
$x+7y+5z-43=0$

6a) $\vec{PQ} = (4,-1,3)$, $\vec{QR} = (-5,2,1)$
$\vec{n} = \vec{PQ} \times \vec{QR} = (-1-6, -15-4, 8-5)$ $R(-2,3,5)$
$\vec{n} = (-7,-19,3)$
$-7x-19y+3z+D=0$
$-7(-2)-19(3)+3(5)+D=0$
$14-57+15+D=0$
$D=28$
$-7x-19y+3z+28=0$
$7x+19y-3z-28=0$

b) The same
c) Because the 3 points lie on the plane.

7. $A(-2,3,1)$ $B(3,4,5)$ $C(1,1,0)$
$\vec{AB} = (5,1,4)$ $\vec{BC} = (-2,-3,-5)$ $\vec{m} = (3,-2,-1)$
$\vec{AB} \times \vec{BC} = (7,17,-13)$
$\therefore \pi: 7x+17y-13z+D=0$
$7(1)+17(1)-13(0)+D=0$
$24+D=0$
$D=-24$
$\pi: 7x+17y-13z-24=0$

8. ① Direction vector from $P(1,3,0)$ to $Q(2,0,1)$
$\vec{PQ} = (1,-3,1)$
② Normal to the plane:
$\vec{PQ} \times \vec{m} = (-20,-9,-7)$
③ Cartesian equation:
$-20x-9y-7z+D=0$
$-20(2)-9(0)-7(1)+D=0$
$-47+D=0$
$D=47$
$\therefore \pi: 20x+9y+7z-47=0$

9a) $\vec{n} = (2,2,-1)$
$|\vec{n}| = \sqrt{2^2+2^2+1^2}$
$=3$
$\hat{n} = \frac{1}{3}(2,2,-1)$

b) $\vec{n} = (4,-3,1)$
$|\vec{n}| = \sqrt{4^2+3^2+1^2}$
$=\sqrt{26}$
$\hat{n} = \frac{1}{\sqrt{26}}(4,-3,1)$

c) $\vec{n} = (3,-4,12)$
$|\vec{n}| = \sqrt{3^2+4^2+12^2}$
$=13$
$\hat{n} = \frac{1}{13}(3,-4,12)$

10. $\vec{AB} = (-1,-1,6)$
$\vec{AB} \times \vec{m} = (-9,15,1)$
$-9x+15y+z+D=0$
$-9(1)+15(1)+5+D=0$
$11+D=0$
$D=-11$
$\therefore \pi: 9x-15y-z+11=0$

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11. Line joining the points
$\vec{m} = (2, -4, -1)$ ← this is also normal to the plane, so use it!
$\pi: 2(1) - 4(2) - 1(1) + D = 0$
$2 - 8 - 1 + D = 0$
$D = 7$
$2x - 4y - z + 7 = 0$

13a) $\vec{n_1} = (1, 2, -3)$ $\vec{n_2} = (1, 2, 0)$
$|\vec{n_1}| = \sqrt{14}$ $|\vec{n_2}| = \sqrt{5}$
$\cos\theta = \frac{1+4}{(\sqrt{14})(\sqrt{5})}$
$= \frac{5}{\sqrt{70}}$
$\theta = 53.3°$

b) $\vec{m} = (-2, 3, 1)$ ← direction vector ⊥ to plane → normal
$-2x + 3y + z + D = 0$
$-2(1) + 3(2) + 1 + D = 0$
$5 + D = 0$
$D = -5$
$2x - 3y - z + 5 = 0$

12a) Find the normal to each plane and then find the angle between them using the formula $\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

b) $\vec{n_1} = (1, 0, -1)$ $\vec{n_2} = (2, 1, -1)$
$|\vec{n_1}| = \sqrt{2}$ $|\vec{n_2}| = \sqrt{6}$
$\cos\theta = \frac{2+1}{(\sqrt{2})(\sqrt{6})}$
$\cos\theta = \frac{3}{2\sqrt{3}}$ or $\frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$
$\boxed{\theta = 30°}$

14a) $n_1 = (4, k, -2)$ ÷2
$n_2 = (2, 4, -1)$
$k = 8$.

b) $\vec{n_1} \cdot \vec{n_2} = 0$
$(4, k, -2) \cdot (2, 4, -1) = 0$
$8 + 4k + 2 = 0$
$k = -5/2$

c) Coincident → intersect at all points
$(k = 8)$
$4x + 8y - 2z + 1 = 0$ ①
$2x + 4y - z + 4 = 0$ ②
These values are not scalar multiples, so they can be parallel, but not coincident
① ÷ 2 = $2x + 4y - z + \frac{1}{2} = 0$
② ⊖ $2x + 4y - z + 4 = 0$
$-3\frac{1}{2} \neq 0$ ∴