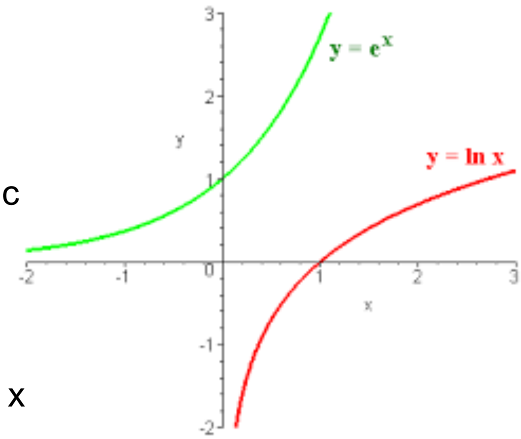
Date: ____________________

Calculus Appendix 3: The Natural Logarithm and its Derivative

The natural logarithm function, $y = \ln x$, is just the inverse of the exponential function $y = e^x$.



Recall that you can write a logarithmic function in exponential form:

$y = \log_b x \longrightarrow b^y = x$

This means that we can rewrite $y = \ln x$ as $x = e^y$. (The base of the natural logarithm is 'e')

We can apply implicit differentiation to determine the derivative of this expression.

Differentiate $x = e^y$

You can now add the derivative of $y = \ln x$ to your derivative tool box. Please remember to apply the derivative rules that you already know when working with composite functions that include the natural logarithm!!

Practice Problems:

1) Find the derivative for each of the following functions:

a) $y = \ln 2x$ b) $y = x^3 \ln x$ c) $y = \ln (x^2-3x)$

2) Determine the equation of a tangent to the curve $y = x^2 \ln x$ at the point (1, 0).

Revisiting Derivatives of Exponential Functions

We saw in chapter 5 that the derivative of $y = a^x$ is $y' = a^x \ln a$. Now that we know how to use implicit differentiation and how to take the derivative of the natural logarithm we can see why this is the case.

Example: Differentiate $y = 2^x$ by writing it in terms of the natural logarithm.