

## Determining the Anti-Derivative of a Function

We find an antiderivative for a function when its derivative is known. For example, if I know the function that represents the velocity of a particle, I would use an antiderivative to find the function that represents the position of that particle.

**Definition:** A function  $F$  is called an antiderivative of  $f$  on an interval if  $F'(x) = f(x)$  for all values of  $x$  in the interval.

You will typically find antiderivatives for functions whose derivatives are found using the power rule, but we could also use it with trig functions and derivatives involving Euler's number. You will not usually use an antiderivative to 'undo' a complex derivative (product rule, quotient rule, chain rule, etc.).

The only catch is that we have to remember that the derivative of a constant is zero. Because of this, we have to add a constant,  $C$ , to our antiderivative.

ex/ Given the function  $f'(x) = x^3$ , determine the antiderivative,  $f(x)$ .

$$F(x) = \frac{1}{4}x^4 + C \quad \text{Check: } F'(x) = x^3 \quad \text{!!}$$

What would we need to know to find a value for  $C$ ? Determine the equation of the antiderivative function if  $F(2) = 7$ .

$$\begin{aligned} 7 &= \frac{1}{4}(2)^4 + C \\ 7 &= 4 + C \\ 3 &= C \end{aligned} \quad F(x) = \frac{1}{4}x^4 + 3$$

**2 Table of Antidifferentiation Formulas**

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$1/x$	$\ln x $	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}x$
$e^x$	$e^x$	$\frac{1}{1+x^2}$	$\tan^{-1}x$
$\cos x$	$\sin x$		

Practice Problems

1. Find  $f$  if  $f'(x) = 4\sin x - 3x^5 + 6\sqrt{x^3}$ .

$$F(x) = -4 \cos x - \frac{1}{2} x^6 + \frac{24}{7} x^{7/4} + C$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{7}{4} b = b$$

$$b = \frac{24}{7}$$

2. Find  $f$  if  $f''(x) = 12x^2 + 6x - 4$ ,  $f(0) = 4$ , and  $f(1) = 1$ .

$$f'(x) = 4x^3 + 3x^2 - 4x + C_1$$

$$f(x) = 1x^4 + x^3 - 2x^2 + C_1x + C_2$$

$$f(0) = 4$$

$$f(1) = 1: 1 = 1 + 1 - 2 + C_1 + C_2$$

$$\therefore C_2 = 4$$

$$-3 = C_1$$

$$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

3. A particle moves in a straight line and has acceleration given by  $a(t) = 6t + 4$ . Its initial velocity is  $v(0) = -6$  cm/s and its initial displacement is  $s(0) = 9$  cm. Find its position function,  $s(t)$ .

$$a(t) = 6t + 4$$

$$s(t) = t^3 + 2t^2 - 6t + C$$

$$v(t) = 3t^2 + 4t + C$$

$$= 3t^2 + 4t - 6$$

$$s(t) = t^3 + 2t^2 - 6t + 9$$

From the practice exercises provided, do #1 - 10, 12, 13, 17, 19 - 22, 25, 26, 29, 34, 35, 37, 59, 61, 63, 64