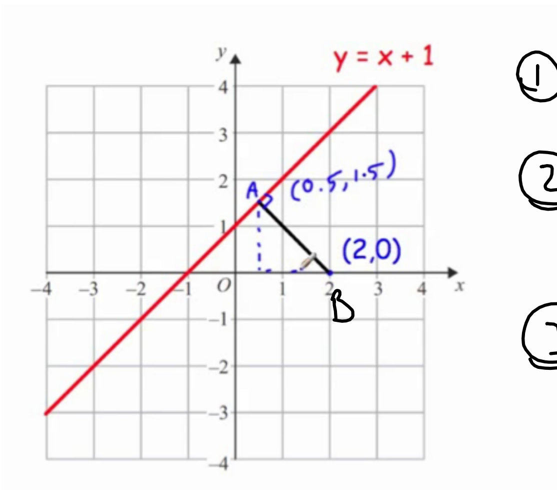
Date: ________________

9.5 The Distance from a Point to a Line in R^2 and R^3

Remember that the distance from a point to a line (or plane) is always the perpendicular distance. In grade ten, we did this in R^2, but it was a long process. Take a minute to try to figure out how you would find the distance from the point to line shown below.

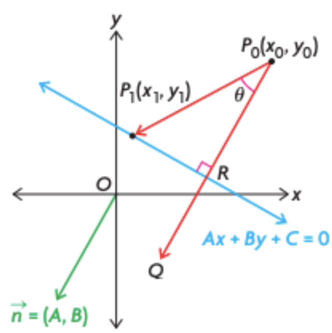


① Equation for AB

② Sub / Elim to get POI (A)

③ Distance formula for AB

Fortunately, now that we have some knowledge of vectors, we can use a formula to find the distance from any point to a line in R^2:



$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

Where (A, B) is the normal vector to the line and (x_o, y_o) is the point given.

A detailed proof of this formula is shown in your text book.

Example: Find the distance from the point P(-3, 5) to the line $4x - y + 3 = 0$.

A B (x_o, y_o)

$$d = \frac{|4(-3) - 1(5) + 3|}{\sqrt{4^2 + 1^2}}$$

$$= \frac{|-14|}{\sqrt{17}}$$

$$d = \frac{14\sqrt{17}}{17} \text{ units}$$

Why won't this formula work in R^3?

No Cartesian equation

Refer to p. 537 for a detailed proof of the formula to find the distance from a point to a line in R^3.

Distance, d, from a Point, P, to the Line $\vec{r} = \vec{r}_0 + s\vec{m}, s \in \mathbf{R}$

In R^3, $d = \dfrac{|\vec{m} \times \vec{QP}|}{|\vec{m}|}$, where Q is a point on the line and P is any other point, both of whose coordinates are known, and $\vec{m}$ is the direction vector of the line.

Example: Determine the distance from the point P(-1, 1, 6) to the line with equation r = (1, 2, -1) + t(0, 1, 1).

Q $\qquad$ $\vec{m}$

$$\begin{array}{cc} 1 & -1 \\ 1 & 7 \\ 0 & -2 \\ 1 & -1 \end{array}$$

$\vec{QP} = (-2, -1, 7)$

$\vec{m} \times \vec{QP} = (7+1, -2, 2)$

$= (8, -2, 2)$

$$d = \frac{|(8, -2, 2)|}{|(0, 1, 1)|}$$

$$= \frac{\sqrt{\cancel{72}}}{\sqrt{2}}$$

$= \sqrt{36}$

$= 6$ units

9.6 The Distance from a Point to a Plane

As long as you have the Cartesian equation for the plane that you are finding the distance to (if you don't have it, you can always find it!), we simply need to adjust the formula to find the distance from a point to a line in R^2.

Distance from a Point $P_0(x_0, y_0, z_0)$ to the Plane with Equation $Ax + By + Cz + D = 0$

In R^3, $d = \dfrac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$, where d is the required distance between the point and the plane.

What is different in this formula?

z-component, so there are more terms

Example: Find the distance from (-5, 2, 8) to the plane $3x - 4y + z - 2 = 0$.

$x_0 \quad y_0 \quad z_0 \qquad A \quad B \quad C \quad D$

$$d = \frac{|3(-5) - 4(2) + 8 - 2|}{\sqrt{3^2 + 4^2 + 1^2}}$$

$$= \frac{|-17|}{\sqrt{26}}$$

$$= \frac{17\sqrt{26}}{26} \text{ units}$$

A trickier example:

Pt-$(0, 0, -2)$

Determine the distance between two parallel planes, $2x - y + 2z + 4 = 0$ and $2x - y + 2z + 16 = 0$.

Find a point on $2x - y + 2z + 4 = 0$

Let $x = 0$, $y = 0$

$2z + 4 = 0$

$z = -2$

$(0, 0, -2)$

$$d = \frac{|(2)(0) - 1(0) + 2(-2) + 16|}{\sqrt{2^2 + 1^2 + 2^2}}$$

$$= \frac{12}{3}$$

$= 4$ units.