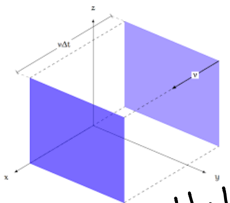
Date: ____________

9.3 The Intersection of Two Planes

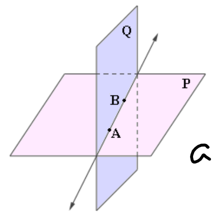
The Intersection of Two Planes

There are three ways that two planes can intersect.



parallel

coincident



a line

What is different about the intersection of planes from that of lines?

- the intersect along a line
- they can't be skew

Identify the type of intersection that occurs between the pairs of planes provided below, and explain how you reached your conclusion.

Elim: $0=0$

a) $2x - y + z = 4$ ①
$4x - 2y + 2z = 8$ ②

b) $6x + 4y - 2z = 8$ ①
$3x + 2y - z = 8$ ②

parallel

①×2: $4x - 2y + 2z = 8$

$2① = ②$ ∴ They are coincident

②×2: $6x + 4y - 2z = 16$

$-$ $6x + 4y - 2z = 8$

$0 = 8$!!

Now let's solve to find the equation of the line of intersection for two planes. We can do this using matrices, or using a series of substitution/elimination steps like we saw yesterday. The key thing to remember is that you need to introduce parameters so that we can develop an equation for a line! There is not a single unique solution.

Example: Determine the solution to x - y + z = 3 and 2x + 2y -2z = 3.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 2 & 2 & -2 & 3 \end{array}\right] \xrightarrow{2R_1 - R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & -4 & 4 & 3 \end{array}\right]$$

work with this

zero here

$-4y + 4z = 3$ Let $y = t$

$-4t + 4z = 3$

$4z = 3 + 4t$

$z = \frac{3}{4} + t$

From ①:

$x - y + z = 3$

$x - t + \frac{3}{4} + t = 3$

$x = 3 - \frac{3}{4}$

$x = \frac{9}{4}$

Parametric equations:

$x = \frac{9}{4}$

$y = t$

$z = \frac{3}{4} + t$