

Date: \_\_\_\_\_

## 9.2 Systems of Equations

### Systems of Equations

We already know how to solve a system of equations in  $R^2$  (substitution, elimination, graphing). We can also use a matrix and elementary row operations to solve in both  $R^2$  and  $R^3$  as long as we have an appropriate number of equations (same # of equations as unknowns).

#### Interpreting an Algebraic Solution Geometrically

Case 1: The solution is  $0 = a$ , where  $a$  is any constant. What does this mean?

$$\left[ \begin{array}{ccc|c} - & - & - & - \\ 0 & 0 & 0 & a \end{array} \right]$$

no solution, no intersection

inconsistent

Case 2: The solution is  $x = a$ ,  $y = a$ , or  $z = a$ , where  $a$  is any constant. What does this mean?

$$\left[ \begin{array}{ccc|c} - & - & - & - \\ 0 & 0 & 0 & a \end{array} \right]$$

one shared POI for all lines/planes

consistent

Case 3: The solution is  $0 = 0$ . What does this mean, and what do we need to do in when this occurs?

$$\left[ \begin{array}{ccc|c} - & - & - & - \\ 0 & 0 & 0 & 0 \end{array} \right]$$

consistent, infinite # of possible solutions

assign parameters to variables

ex/ Let  $z = t$

**Vocabulary Note:** A system of equations is consistent if it has a solution (one POI or an infinite number of POIs), and it is inconsistent if it does not have a solution (parallel lines or planes).

If you really dislike matrices, you can solve using elementary operations (elimination/substitution). We will do one example this way, but I strongly recommend using matrices!!

Example: Solve the system of equations below using elementary operations.

①  $x - y + z = 1$

②  $2x + y - z = 11$

③  $3x + y + 2z = 12$

Sub into ①

$$4 - y - 1 = 1$$

$$-y = -2$$

$$\boxed{y = 2}$$

POI

$$(4, 2, -1)$$

① + ③:  $3x = 12$

$$\boxed{x = 4}$$

Check in ② + ③

$$2(4) + 2 + 1 \stackrel{?}{=} 11$$

$$8 + 3 = 11 \checkmark$$

② - ②:  $-x - 3z = -1$

Sub in  $x = 4$   $-4 - 3z = -1$

$$-3z = 3$$

$$\boxed{z = -1}$$

$$3(4) + 2 - 2 \stackrel{?}{=} 12$$

$$12 = 12 \checkmark$$

