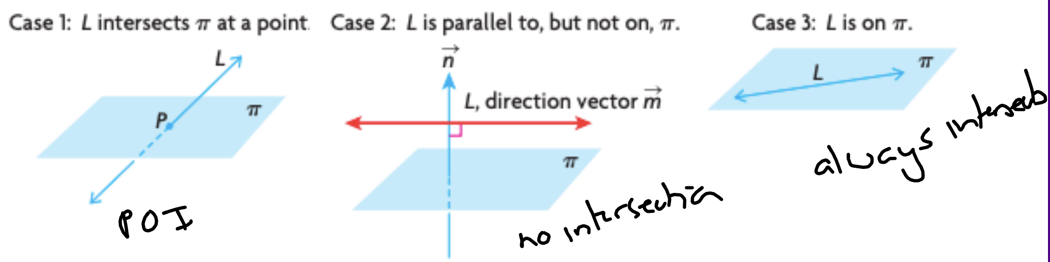
Date: ____________

9.1 The Intersection of a Line with a Plane and the Intersection of Two Lines

1) Intersection of a Line with a Plane

A line, L, can intersect with a plane, π, in three ways (see supplemental video for models):



Is it possible for a line to have a finite number (greater than one) of intersection points with a plane? Explain.

No → it can cross through once or an infinite # of times

Let's use what we know about equations of lines and planes to determine the point of intersection of a line and a plane.

Example: Determine the point of intersection between the line L: $\vec{r} = (3, 1, 2) + t(1, -4, -8)$, $t \in R$, and the plane π: $4x + 2y - z - 8 = 0$.

① Parametric equations

$x = 3 + t$

$y = 1 - 4t$

$z = 2 - 8t$

② Sub into π

$4(3+t) + 2(1-4t) - (2-8t) - 8 = 0$

$12 + 4t + 2 - 8t - 2 + 8t - 8 = 0$

$4t + 4 = 0$

$t = -1$

③ use $t = -1$ to find (x, y, z)

$x = 3 - 1$
$= 2$

$y = 1 - 4(-1)$
$= 5$

$z = 2 - 8(-1)$
$= 10$

POI $(2, 5, 10)$

What would expect to get as an algebraic solution if the line is parallel to the plane? If the line was part of the plane?

$0 = \#$ $\qquad$ $0 = 0$

Example: Determine the point(s) of intersection of the line

L: $\vec{r} = (3, -2, 1) + s(14, -5, -3)$ and the plane $x + y + 3z - 4 = 0$, if any exist.

$x = 3 + 14s$

$y = -2 - 5s$

$z = 1 - 3s$

$3 + 14s - 2 - 5s + 3(1 - 3s) - 4 = 0$

$4 + 9s - 9s - 4 = 0$

$\boxed{0 = 0}$

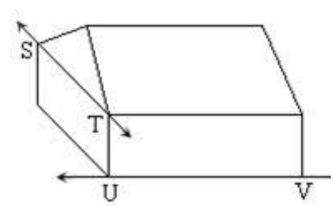
$\therefore$ The line is on the plane.

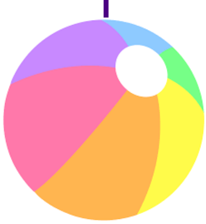
2) <u>The Intersection of Two Lines in R^2 and R^3</u>

We have covered the ways that lines in R^2 can intersect in great detail in grade ten. Remember that they can have one distinct point of intersection (different slopes), no points of intersection (parallel lines), or an infinite number of points of intersection (same line).

Things are a little bit different in R^3. These three cases still exist (a point, all of the points, no points), but there is one new case that occurs called skew lines.

Skew lines - Lines that are not parallel, but that also do not intersect (non - coplanar)





When you are determining the point (or type) of intersection for lines in R^3, please be sure to check the direction vectors. If they are scalar multiples of one another, we know that they are parallel. What does this tell you about the solution to the pair of equations?

Practice Problems

1) Find the point of intersection for L_1: r = (-3, 1, 4) + s(-1, 1, 4) and L_2: r = (1, 4, 6) + t(-6, -1, 6).

① Parametric Equations

$x_1 = -3 - s$ $\quad x_2 = 1 - 6t$

$y_1 = 1 + s$ $\quad y_2 = 4 - t$

$z_1 = 4 + 4s$ $\quad z_2 = 6 + 6t$

② Systems of Equations

For x: $-3 - s = 1 - 6t$ For y:

① $s - 6t = -4$ $\quad 1 + s = 4 - t$

② $s + t = 3$ $\quad$ ② $s + t = 3$

①−②: $-7t = -7$ $\quad s + 1 = 3$

$\boxed{t = 1}$ $\quad \boxed{s = 2}$

③ Find POI

$x_1 = -5$ $\quad x_2 = -5$ ☺

$y_1 = 3$ $\quad y_2 = 3$ ☺

$z_1 = 12$ $\quad z_2 = 12$ ☺

POI: $(-5, 3, 12)$

2) Show that the lines given below are skew lines. Explain how you know.

L_1: r = (9, 1, 2) + s(5, 0, 4)

L_2: r = (8, 2, 3) + t(4, 1, -2) ← not parallel

$x_1 = 9 + 5s$ $\quad x_2 = 8 + 4t$

$y_1 = 1$ $\quad y_2 = 2 + t$ → $1 = 2 + t$, $-1 = t$

$z_1 = 2 + 4s$ $\quad z_2 = 3 - 2t$

Use $t = -1$ to solve for s in both x + z.

For x:

$8 + 4t = 9 + 5s$

$4 = 9 + 5s$

$-5 = 5s$

$-1 = s$

For z:

$2 + 4s = 3 - 2t$

$2 + 4s = 5$

$4s = 3$

$s = 3/4$

$-1 \neq 3/4$

not the same, so the lines are skew.