Date: ________________

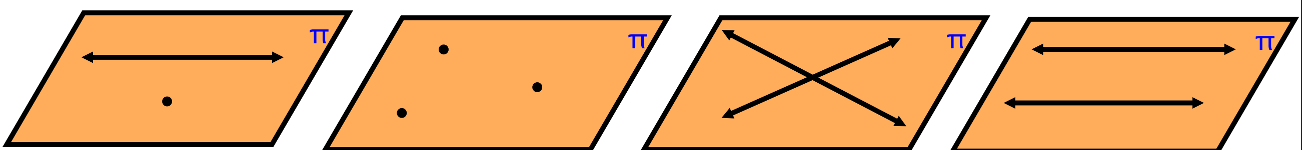
8.4 Vector and Parametric Equations of a Plane

plane - a flat surface that extends infinitely far in all directions;
represented by parallelograms labelled with π

Two non-collinear vectors form a basis for a plane, but there are an infinite number of planes parallel to that specific plane. We also require a point on the plane to be able to find the plane that we are interested in.

We can determine an equation for a specific plane if we have:

- the equation of a line on the plane and a point on the plane (1)
- three non - collinear points on the plane (2)
- the equations of two intersecting lines on the plane (3)
- the equations of two parallel and non-coincident lines on the plane (4)



(1) (2) (3) (4)

Linear Combinations and Planes

This is where linear combinations and spanning sets are important! They will allow us to understand how to get the equation of a plane.

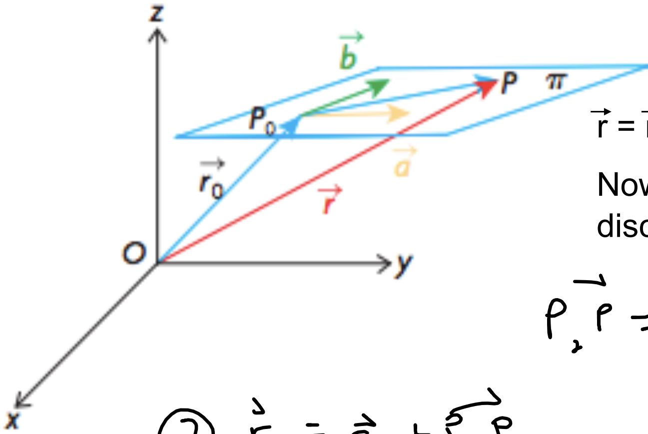
Recall from Chapter 6:

- Any two non-collinear vectors, $\vec{a}$ and $\vec{b}$, span a plane in R^3.
- Any vector, $\vec{c}$, in the plane can be expressed as a linear combination of $\vec{a}$ and $\vec{b}$.

$s(a_1, a_2, a_3) + t(b_1, b_2, b_3) = (c_1, c_2, c_3)$, where s and t are real numbers

The equation of a plane considers all of the possible linear combinations of these vectors. Remember that $\vec{a}$ and $\vec{b}$ on their own can create an infinite number of parallel planes, so it is important that we know the coordinates of a point on the plane as well!

The Vector Equation of a Plane



$\vec{r} = \vec{r}_o + s\vec{a} + t\vec{b}$, s, t are real numbers

Now let's look at the diagram and discuss where this came from...

$\overrightarrow{P_0P} = s\vec{a} + t\vec{b}$ ①

② $\vec{r} = \vec{r}_0 + \overrightarrow{P_0P}$

$\therefore \vec{r} = \vec{r}_0 + s\vec{a} + t\vec{b}$

Example: Write the vector equation of a plane with direction vectors $\vec{a} = (-2, 3, 5)$ and $\vec{b} = (4, 1, 7)$ that passes through the point (-3, 2, 4).

$$\vec{r} = (-3, 2, 4) + s(-2, 3, 5) + t(4, 1, 7)$$

Food for Thought:

Is a plane two-dimensional or three - dimensional?

2-D

How can we find more points on a plane once we have the vector equation?

assign values for s and t.

What do you think that the parametric equations of a plane would look like?

$x = x_0 + sa_1 + tb_1$

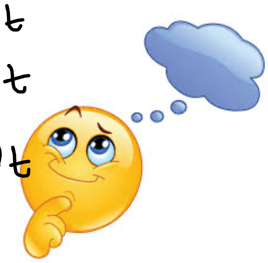
$y = y_0 + sa_2 + tb_2$

$z = z_0 + sa_3 + tb_3$

$x = -3 - 2s + 4t$

$y = 2 + 3s + t$

$z = 4 + 5s + 7t$



Practice Problems

1. Determine the vector and parametric equations for a plane that contains the points A(-1, 3, 8), B(-1, 1, 0) and C(4, 1, 1). Does the point (14, 1, 3) lie on this plane?

$\vec{r} = (-1,1,0) + s(0,-2,-8) + t(5,-2,-7)$

$\vec{AB} = (0,-2,-8)$

$\vec{AC} = (5,-2,-7)$

$x = -1+5t \qquad y = 1-2s-2t \qquad z = -8s-7t$

$14 = -1+5t \qquad 1 = 1-2s-2t \qquad 3 = -8s-7t$

$\frac{15}{5} = \frac{5t}{5} \qquad 0 = -2s-2t \qquad 3 \overset{?}{=} 24-21$

$3 = t \qquad 2s = -2t \qquad 3 = 3$:)

$s = -t$

$s = -3$

$\therefore (14,1,3)$ is on the plane.

2. Determine the vector and parametric equations of the plane containing the point P(-1, 3, 7) and the line L: $\vec{r}$ = (1, 2, 1) + s(-2, 3, 1).

$\vec{a} = (-2,3,1)$

$\vec{b} = (2,-1,-6)$

$\pi: \vec{r} = (1,2,1) + s(-2,3,1) + t(2,-1,-6)$

$x = 1-2s+2t \qquad y = 2+3s-t \qquad z = 1+s-6t$

3. Determine the coordinates of the point where the plane with equation $\vec{r}$ = (4, 1, 6) + s(10, 8, -1) + t(-5, 1, -3) crosses the y axis.

$P: (0, y, 0)$

$x = 4+10s-5t$

$0 = 4+10s-5t$

$-4 = 10s-5t$

$-4 = 10(6-3t)-5t$

$-4 = 60-30t-5t$

$-64 = -35t$

$\frac{64}{35} = t$

$z = 6-s-3t$

$0 = 6-s-3t$

$s+3t = 6$

$s = 6-3t$

$s = 6-3\left(\frac{64}{35}\right)$

$s = \frac{18}{35}$

$y = 1+8s+t$

$y = 1+8\left(\frac{18}{35}\right) + \frac{64}{35}$

$= \frac{243}{35}$

$\therefore P\left(0, \frac{243}{35}, 0\right)$