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### 8.3 Vector, Parametric, and Symmetric Equations of a Line in $\mathbb{R}^3$

The equations for a line in  $\mathbb{R}^3$  are very similar to those for a line in  $\mathbb{R}^2$ . The only difference is that they have a  $z$  - coordinate for points and a  $z$  - component for vectors. Vector and parametric equations are required in three space because slope is not defined.

Vector Equation of a Line in  $\mathbb{R}^3$ :

$$\vec{r} = (x_0, y_0, z_0) + t(a, b, c)$$

where  $(x_0, y_0, z_0)$  is any point on the line and  $(a, b, c)$  is the direction vector for the line.

Example: Find the vector equation of a line through the point  $A(1, 3, 6)$  with direction vector  $\vec{m} = (1, 7, 3)$

Example: Determine a vector equation for a line through the points  $A(-1, 3, -5)$  and  $B(2, -1, 4)$ .

Parametric Equations of a Line in  $\mathbb{R}^3$ :

$$\begin{aligned}x &= x_0 + ta \\y &= y_0 + tb \\z &= z_0 + tc\end{aligned}$$

Symmetric Equations of a Line in  $\mathbb{R}^3$ :

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} (= t)$$

\*a, b, and c cannot equal zero!

Example: Write the parametric and symmetric equations for the line in the previous example.

Use the general form of the parametric equations for a line in three-space to derive the general form of the symmetric equation for the line.

