Date: ________________

8.3 Vector, Parametric, and Symmetric Equations of a Line in R^3

The equations for a line in R^3 are very similar to those for a line in R^2. The only difference is that they have a z - coordinate for points and a z - component for vectors. Vector and parametric equations are required in three space because slope is not defined.

Vector Equation of a Line in R^3:

$$\vec{r} = (x_o, y_o, z_o) + t(a, b, c)$$

where (x_o, y_o, z_o) is any point on the line and (a, b, c) is the direction vector for the line.

Example: Find the vector equation of a line through the point A(1, 3, 6) with direction vector $\vec{m} = (1, 7, 3)$

$$\vec{r} = (1, 3, 6) + t(1, 7, 3)$$

Example: Determine a vector equation for a line through the points A(-1, 3, -5) and B(2, -1, 4).

$\vec{AB} = (3, -4, 9)$

$\vec{d} = (3, -4, 9)$

$\vec{r} = (-1, 3, -5) + t(3, -4, 9)$

Parametric Equations of a Line in R^3:	Symmetric Equations of a Line in R^3:
$x = x_o + ta$ $y = y_o + tb$ $z = z_o + tc$	$\frac{x - x_o}{a} = \frac{y - y_o}{b} = \frac{z - z_o}{c}$ (= t) *a, b, and c cannot equal zero!

Example: Write the parametric and symmetric equations for the line in the previous example.

$x = -1 + 3t \rightarrow t = \frac{x+1}{3}$

$y = 3 - 4t \rightarrow t = \frac{y-3}{-4}$

$z = -5 + 9t \rightarrow \frac{z+5}{9} = t$

$\frac{x+1}{3} = \frac{-(y-3)}{4} = \frac{z+5}{9}$

Symmetric equations

Use the general form of the parametric equations for a line in three-space to derive the general form of the symmetric equation for the line.

$x = x_o + ta \quad y = y_o + tb \quad z = z_o + tc$

$\frac{x - x_o}{a} = t \quad \frac{y - y_o}{b} = t \quad \frac{z - z_o}{c} = t$

$$\frac{x - x_o}{a} = \frac{y - y_o}{b} = \frac{z - z_o}{c}$$