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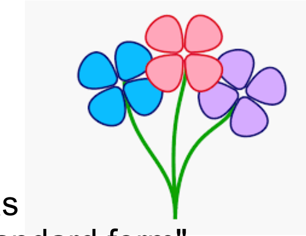


8.2 Cartesian Equation of a Line

The scalar or Cartesian equation of a line in a plane has the form Ax + By + C = 0. Until now, this was called "standard form".

The Cartesian equation can be determined from either the vector or parametric equation in a number of ways.

Example: Find the Cartesian equation for the line $\vec{r}$ = (2, -1) + t(4, 3).

Method 1: Using slope (direction vector) and a point.

$m = \frac{3}{4}$

$y = mx + b$

$-1 = \frac{3}{4}(2) + b$

$-1 = \frac{3}{2} + b$

$b = -\frac{5}{2}$

$4(y) = 4\left(\frac{3}{4}x - \frac{5}{2}\right)$

$4y = 3x - 10$

$3x - 4y - 10 = 0$

Method 2: Using parametric equations.

$x = 2 + 4t \qquad y = -1 + 3t$

$\frac{x-2}{4} = t \qquad \frac{y+1}{3} = t$

$\frac{x-2}{4} = \frac{y+1}{3}$

$3x - 6 = 4y + 4$

$3x - 4y - 10 = 0$

The scalar equation can also be determined by using a vector that is perpendicular to the line. This is called a normal vector, $\vec{n}$.

Method 3: Using a normal vector.

$\vec{d} = (4, 3)$

$\vec{n} = (-3, 4)$

(A, B)

$Ax + By + C = 0$

$-3x + 4y + C = 0$

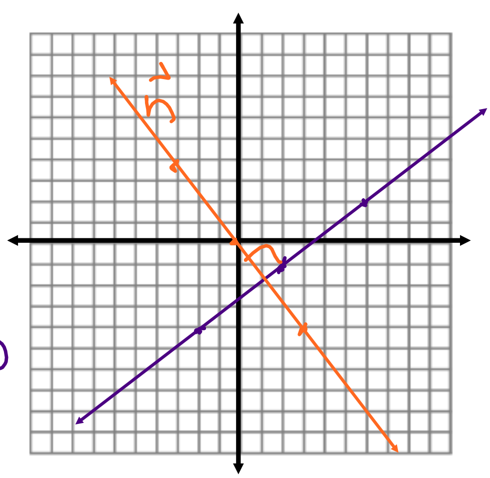
$-3(2) + 4(-1) + C = 0$

$-6 - 4 + C = 0$

$C = 10$

$-3x + 4y + 10 = 0$

$3x - 4y - 10 = 0$



General Case:

Find the Cartesian equation of a line with normal n = (A, B) which passes through the point $P(x_o, y_o)$.

$$Ax_o + By_o + C = 0$$

$$C = -Ax_o - By_o$$

$$\therefore Ax + By - (Ax_o + By_o) = 0$$

Example: Determine a Cartesian equation for a line through $P_o(-1, 2)$ and parallel to the line with parametric equations x = 1 + 2t and y = 2 - 3t.

$\vec{d} = (2, -3)$

$\vec{n} = (3, 2)$

A B

$3(-1) + 2(2) + C = 0$

$-3 + 4 + C = 0$

$C = -1$

$\boxed{3x + 2y - 1 = 0}$

Finding a Vector Equation Given a Cartesian Equation

Remember that A and B are the components of the vector that is normal to the line. The dot product of the normal and the direction vector must be:

zero!

Example: Find a vector equation for the line with Cartesian equation 3x - 2y + 6 = 0. Let $x = 0$, $y = 3$

$\vec{n} = (3, -2)$

$\vec{d} = (2, 3)$

$\vec{n} \cdot \vec{d} = 6 - 6 = 0$:)

$\vec{r} = (x_o, y_o) + t(2, 3)$

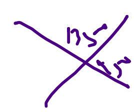
$\vec{r} = (0, 3) + t(2, 3)$

Finding the Angle Between Two Lines in R^2

Use what you already know about vectors to determine the acute angle formed at the point of intersection of the vectors provided below.

L_1: (x, y) = (2, 4) + s(-1, 4)

L_2: (x, y) = (5, -2) + t(3, 5)



$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-3 + 20}{(\sqrt{17})(\sqrt{34})}$$

$$\theta = 45°$$

$\vec{a} = (-1, 4)$

$|\vec{a}| = \sqrt{17}$

$\vec{b} = (3, 5)$

$|\vec{b}| = \sqrt{34}$