Date: ____________________

8.1 Vector and Parametric Equations of a Line in R^2

We already know some stuff about forms of equations of lines.

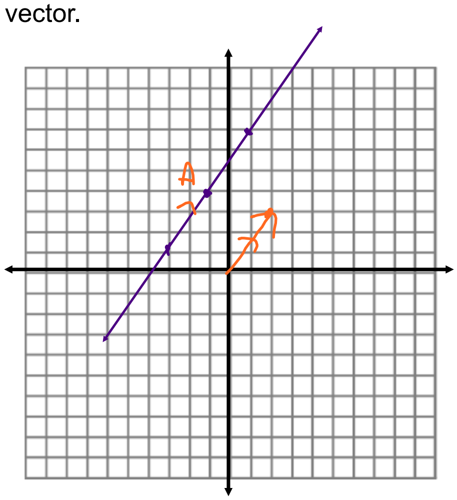
Slope-y-intercept form: $y = mx + b$

Standard Form: $Ax + By + C = 0$

Any point on the line will satisfy any form of the equation for that line. These forms only work in two dimensions though. In order to extend equations into three space, we need to look at new forms of the equation of a line.

1) Vector Equation of a Line in a Plane

If we have a line with a slope of 3/2 that passes through A(-1, 4), we can also describe it as a line that moves in the same direction as the direction vector $\vec{d} = (2, 3)$. Any scalar multiple of d is also a direction vector.



Vector form:

$\vec{r} = (-1, 4) + t(2, 3)$

In general, the vector equation of a line in a plane is:

$$\vec{r} = (x_o, y_o) + t(a, b)$$

where $\vec{r} = (x, y)$ is the position vector of any point on the line, (x_o, y_o) is the position vector of a particular point on the line, and (a, b) is the direction vector, and $t \in R$.

Example: Determine the vector equation of a line with a slope of 3 that passes through the point (2, -5).

$m = \frac{3}{1}$

$\vec{r} = (2, -5) + t(1, 3)$

$\vec{d} = (1, 3)$

Note: If you are asked to identify if lines are coincident, you are just checking to see if their direction vectors are parallel.

2) Parametric Equations of a Line

Consider the vector equation $(x, y) = (x_o, y_o) + t(a, b)$. From this, we can see that $x = x_o + ta$ and $y = y_o + tb$. These are called the parametric equations of the line.

Example: Write the equation of the line from the previous example in parametric form.

$\vec{r} = (2, -5) + t(1, 3)$

$x = 2 + t$

$y = -5 + 3t$

The value of t is called a parameter. This is just an independent variable whose value may be chosen. The values of x and y depend on the parameter that is chosen. Changing the value of 't' changes the magnitude of the direction vector and produces the coordinate of another point on the direction vector.

Example: Find the vector and parametric equations of a line that passes through (4, 6) that is perpendicular to $\vec{r} = (2, 1) + t(8, -4)$.

Point

$m_2 = \frac{2}{1}$

$\vec{d}_2 = (1, 2)$

$\vec{r} = (4, 6) + t(1, 2)$

$x = 4 + t$

$y = 6 + 2t$

$\vec{d} = (8, -4)$

$m = \frac{-4}{8} = -\frac{1}{2}$

Practice Problems:

1. Write the vector and parametric equations for the line containing P(-1, 5) and Q(6, 11). Where does this line cross the x - axis? $R(x, 0)$

$\vec{d} = \vec{PQ}$

$= (6+1, 11-5)$

$= (7, 6)$

$\vec{r} = (-1, 5) + t(7, 6)$

$x = -1 + 7t \rightarrow x = -1 + 7t$

$y = 5 + 6t$

$0 = 5 + 6t$

$-\frac{5}{6} = t$

$= -1 + 7\left(-\frac{5}{6}\right)$

$= -\frac{6}{6} - \frac{35}{6}$

$x = -\frac{41}{6}$ (x-int.)

2. A line is defined by the parametric equations $x = -3 - t$ and $y = 5 + 4t$. Does R(4, 21) lie on this line?

$x = -3 - t$

$4 = -3 - t$

$7 = -t$

$-7 = t$

$y = 5 + 4t$

$21 = 5 + 4t$

$16 = 4t$

$4 = t$

$-7 = t \Longleftrightarrow 4 = t$

not the same, so R is not on the line.