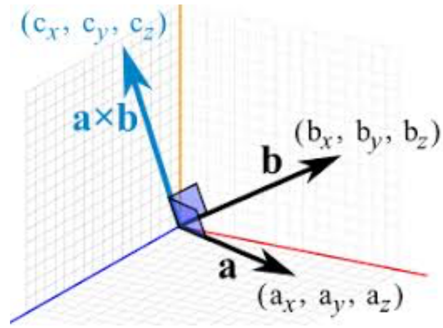
Date:

7.6 The Cross Product of Two Vectors



The cross product of two vectors, $\vec{a}$ and $\vec{b}$, in R^3 is defined as the vector that is perpendicular to both a and b. This is also referred to as the vector product because it results in a vector quantity, unlike the scalar dot product.

Formula for the Magnitude of the Cross Product of Geometric Vectors

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$ (often used in physics when you know the angle between two vectors.)

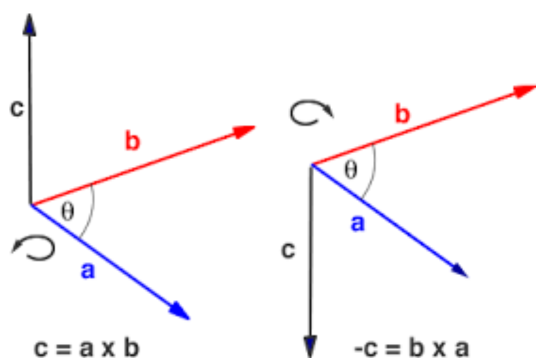
Formula for the Cross Product of Algebraic Vectors (More important!!)

$k(a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$ is a vector perpendicular to both a and b, when $k \in R$.

If k = 1, then $\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$
If k = -1, then $\vec{b} \times \vec{a} = (b_2a_3 - b_3a_2, b_3a_1 - b_1a_3, b_1a_2 - b_2a_1)$

There are an infinite number of vectors that are perpendicular to a and b, but they are all scalar multiples of one another. The simplest values of k to work with are 1 and -1, so our formula is based upon this assumption. A detailed development of this formula is presented on p. 403 on your text book.

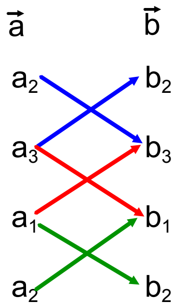
The cross product is NOT commutative! $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$!



The formula for calculating the cross product of algebraic vectors is not easy to remember. Use the following trick so that you don't have to memorize the formula!

For $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$,

1) List your components in the order shown below.



2) Multiply in the direction of the arrows.

$x = a_2b_3 - a_3b_2$

$y = a_3b_1 - a_1b_3$

$z = a_1b_2 - a_2b_1$

3) $\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1)$

<u>Example</u>: For vectors $\vec{a} = (-2, 3, 6)$ and $\vec{b} = (4, 1, 5)$, determine $\vec{a} \times \vec{b}$ and $|\vec{a} \times \vec{b}|$.

Use the dot product to check your answer. What are you expecting?

Properties of the Cross Product

Let $\vec{p}$, $\vec{q}$, and $\vec{r}$ be three vectors in R^3, and let $k \in \mathbf{R}$.

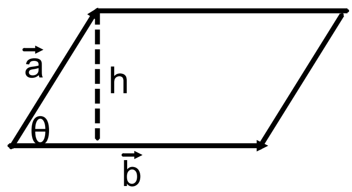
Vector multiplication is not commutative: $\vec{p} \times \vec{q} = -(\vec{q} \times \vec{p})$,

Distributive law for vector multiplication: $\vec{p} \times (\vec{q} + \vec{r}) = \vec{p} \times \vec{q} + \vec{p} \times \vec{r}$,

Scalar law for vector multiplication: $k(\vec{p} \times \vec{q}) = (k\vec{p}) \times \vec{q} = \vec{p} \times (k\vec{q})$, Area of a Parallelogram - A Geometric Application of the Cross Product

For a parallelogram with sides $\vec{a}$ and $\vec{b}$, the area is $|\vec{a} \times \vec{b}|$.

Proof:



Example: Find the area of the parallelogram formed by $\vec{a} = (1, 2, 4)$ and $\vec{b} = (3, 1, 4)$.