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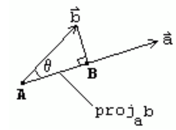
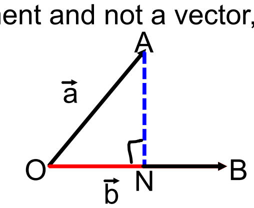


7.5 Scalar and Vector Projections

The projection of $\vec{a}$ on $\vec{b}$ is a line segment produced by drawing a line from the tip of $\vec{a}$ that is perpendicular to $\vec{b}$. Because the projection (ON) is a line segment and not a vector, it is called the scalar projection of $\vec{a}$ on $\vec{b}$.



ON is the scalar projection of $\vec{a}$ on $\vec{b}$.

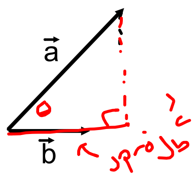
Notation:

sproj ($\vec{a}$ on $\vec{b}$)

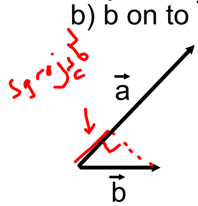
$\text{sproj}_{\vec{b}}\vec{a}$

Example: Draw the following scalar projections.

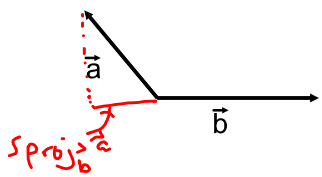
a) $\vec{a}$ on to $\vec{b}$



b) $\vec{b}$ on to $\vec{a}$



c) $\vec{a}$ on to $\vec{b}$



What information do we need to be able to calculate the length of a scalar projection?

· $|\vec{a}|$ (or the vector being projected)

· the angle between the vectors

$\cos\theta = \dfrac{\text{sproj}_{\vec{b}}\vec{a}}{|\vec{a}|}$

$\boxed{\text{sproj}_{\vec{b}}\vec{a} = |\vec{a}|\cos\theta}$

Example: Calculate the scalar projections of $\vec{a}$ on $\vec{b}$ and $\vec{b}$ on $\vec{a}$ for vectors $\vec{a} = (-3, 4, 5)$ and $\vec{b} = (-2, 2, 1)$.

$|\vec{a}| = \sqrt{(-3)^2+4^2+5^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$|\vec{b}| = \sqrt{2^2+2^2+1^2}$

$= 3$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\text{sproj}_{\vec{b}}\vec{a} = \cancel{|\vec{a}|}\left(\dfrac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}|\vec{b}|}\right)$

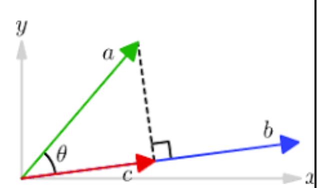
$\boxed{\text{sproj}_{\vec{b}}\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}}$

$\text{sproj}_{\vec{b}}\vec{a} = \dfrac{(-3)(-2)+(4)(2)+(5)(1)}{3}$

$= \dfrac{19}{3}$

$\text{sproj}_{\vec{a}}\vec{b} = \dfrac{19}{5\sqrt{2}}$

$= \dfrac{19\sqrt{2}}{10}$



Direction Angles of a Vector in R^3

Scalar projections can be used to determine the angles that a vector makes with each of the coordinate axes. These are called direction angles, and their corresponding cosine ratios are called direction cosines. You are projecting the vector in three space on to each of the coordinate axes.

Given vector $\overrightarrow{OP} = (a, b, c)$, you can find direction cosines by calculating the scalar projection of OP with each axis. $\vec{i} = (1,0,0)$, $\vec{j} = (0,1,0)$, $\vec{k} = (0,0,1)$

Let α, β, and γ be the angles that $\overrightarrow{OP}$ makes with the positive x, y, and z axis, respectively.

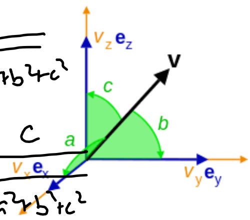
$\cancel{|\overrightarrow{OP}|}\cos\alpha = \dfrac{\overrightarrow{OP}\cdot\vec{i}}{|\overrightarrow{OP}||\vec{i}|}$

$\cos\alpha = \dfrac{(a,b,c)\cdot(1,0,0)}{\sqrt{a^2+b^2+c^2}}$

$\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2+c^2}}$

$\cos\beta = \dfrac{b}{\sqrt{a^2+b^2+c^2}}$

$\cos\gamma = \dfrac{c}{\sqrt{a^2+b^2+c^2}}$



Example: Determine the direction cosines for the vector $\vec{a} = (3, 5, -2)$

$|\vec{a}| = \sqrt{3^2+5^2+(-2)^2}$
$= \sqrt{58}$

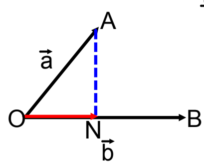
$\cos\alpha = \dfrac{3}{\sqrt{58}} = \dfrac{3\sqrt{58}}{58}$ $\quad\cos\beta = \dfrac{5}{\sqrt{58}} = \dfrac{5\sqrt{58}}{58}$ $\quad\cos\gamma = \dfrac{-2}{\sqrt{58}} = -\dfrac{\sqrt{58}}{29}$

What would you do to find the direction angles?

Take the inverse of cosine.

Vector Projections

A vector projection of a on $\vec{b}$ is just the scalar projection multiplied by $\dfrac{\vec{b}}{|\vec{b}|}$, which is a unit vector pointing in the direction of $\vec{b}$.



Note that $\overrightarrow{ON}$ now has direction indicated.

$\text{sproj}_{\vec{b}}\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ $\quad \text{proj}_{\vec{b}}\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\left(\dfrac{\vec{b}}{|\vec{b}|}\right)$

$\overrightarrow{ON}$ is the vector projection of $\vec{a}$ on $\vec{b}$. Notation is the same, but with a v.

Example: Find the vector projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} = (3, -2, 4)$ and $\vec{b} = (-1, 5, 2)$.

$\text{proj}_{\vec{b}}\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$

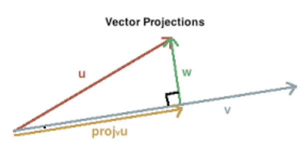
$\text{proj}_{\vec{b}}\vec{a} = \dfrac{3(-1)+(-2)(5)+4(2)}{(-1)^2+5^2+2^2}\cdot\vec{b}$

$= \dfrac{-5}{30}\vec{b}$

$= -\dfrac{1}{6}\vec{b}$

$= -\dfrac{1}{6}(-1,5,2)$

$= \left(\dfrac{1}{6}, -\dfrac{5}{6}, -\dfrac{1}{3}\right)$



A good way to think of the projection of **u** on **v** is as the vector that is the "shadow" that **u** produces on **v**.