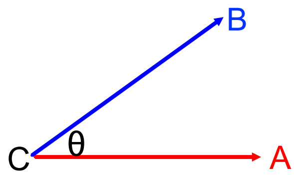
Date: ____________________

7.3 The Dot Product of Two Geometric Vectors

Recall that geometric vectors do not have a coordinate system associated with them, and that the angle between two geometric vectors is assumed to be the angle between the vectors when they are joined tail to tail.

The dot product for any two geometric vectors is defined as the product of their magnitudes multiplied by the cosine of the angle between them.

The Dot Product of Two Vectors



$\vec{CA} \cdot \vec{CB} = |\vec{CA}|\,|\vec{CB}| \cos \theta,\ 0 \leq \theta \leq 180^\circ$

Is the dot product a vector or scalar quantity?

scalar

Use your knowledge of trigonometry to decide when you would expect the dot product to be positive and when you would expect it to be negative.

$|\vec{CA}||\vec{CB}| \rightarrow$ always positive

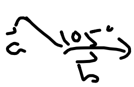
$\cos\theta \rightarrow (+)$ if θ is $0 \leq \theta \leq 90$

$\cos\theta \rightarrow (-)$ if $90^\circ < \theta \leq 180^\circ$

What happens to the dot product if the vectors are perpendicular to each other?

$\cos 90^\circ = 0$, so the dot product of $\perp$ vectors is zero.

Example: Two vectors $\vec{a}$ and $\vec{b}$ have magnitudes 4 and 7, respectively. There is an angle of 105° between them. Calculate $\vec{a} \cdot \vec{b}$.



$\vec{a} \cdot \vec{b} = (4)(7)\cos 105^\circ$

$= -7.25$

What do you think would happen if we took the dot product of a vector and itself?

ex/ If $|\vec{a}| = \sqrt{5}$, calculate $\vec{a} \cdot \vec{a}$.

$\cos 0 = 1$

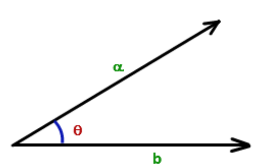
$\vec{a} \cdot \vec{a} = |\vec{a}||\vec{a}|\cos\theta$

$= |\vec{a}|^2 \cos 0^\circ$

$= |\vec{a}|^2$

$= (\sqrt{5})^2$

$= 5$



a.b = |a| |b| Cosθ

Other Properties of the Dot Product

Many of the properties of vector addition and scalar multiplication apply to the dot product.

Properties of the Dot Product

Commutative Property: $\vec{p}\cdot\vec{q} = \vec{q}\cdot\vec{p}$,

Distributive Property: $\vec{p}\cdot(\vec{q} + \vec{r}) = \vec{p}\cdot\vec{q} + \vec{p}\cdot\vec{r}$,

Magnitudes Property: $\vec{p}\cdot\vec{p} = |\vec{p}|^2$,

Associative Property with a scalar K: $(k\vec{p})\cdot\vec{q} = \vec{p}\cdot(k\vec{q}) = k(\vec{p}\cdot\vec{q})$

Remember that the dot product is a scalar quantity, so it makes sense that we are able to apply many of the same properties that we apply to ordinary arithmetic!

Dot product = 0

Practice Problems

1) If the nonzero vectors $\vec{a} + 2\vec{b}$ and $3\vec{a} - \vec{b}$ are perpendicular, and $|\vec{a}| = 2|\vec{b}|$, determine the angle between $\vec{a}$ and $\vec{b}$.

$(\vec{a} + 2\vec{b})\cdot(3\vec{a} - \vec{b}) = 0$

$3\vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + 6\vec{a}\cdot\vec{b} - 2\vec{b}\cdot\vec{b} = 0$

$3|\vec{a}|^2 + 5\vec{a}\cdot\vec{b} - 2|\vec{b}|^2 = 0$

$3(4|\vec{b}|^2) + 5|\vec{a}||\vec{b}|\cos\theta - 2|\vec{b}|^2 = 0$

$12|\vec{b}|^2 - 2|\vec{b}|^2 + 5(2|\vec{b}|)(|\vec{b}|)\cos\theta = 0$

$10|\vec{b}|^2 + 10|\vec{b}|^2\cos\theta = 0$

$\cos\theta = \dfrac{-10|\vec{b}|^2}{10|\vec{b}|^2}$

$\vec{a}\cdot\vec{a} = |\vec{a}|^2$

$\vec{b}\cdot\vec{b} = |\vec{b}|^2$

$(|\vec{a}|)^2 = (2|\vec{b}|)^2$

$|\vec{a}|^2 = 4|\vec{b}|^2$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore \cos\theta = -1$, so θ is $180°$.

2) Use the properties of the dot product to simplify the expression below.

$$3\vec{a}\cdot(\vec{a} - 2\vec{b}) - (\vec{a} + 3\vec{b})\cdot(-2\vec{a} + 5\vec{b})$$

$= 3\vec{a}\cdot\vec{a} - 6\vec{a}\cdot\vec{b} - [-2\vec{a}\cdot\vec{a} + 5\vec{a}\cdot\vec{b} - 6\vec{a}\cdot\vec{b} + 15\vec{b}\cdot\vec{b})$

$= 3|\vec{a}|^2 - 6\vec{a}\cdot\vec{b} + 2|\vec{a}|^2 - 5\vec{a}\cdot\vec{b} + 6\vec{a}\cdot\vec{b} - 15|\vec{b}|^2$

$= 5|\vec{a}|^2 - 5\vec{a}\cdot\vec{b} - 15|\vec{b}|^2$