

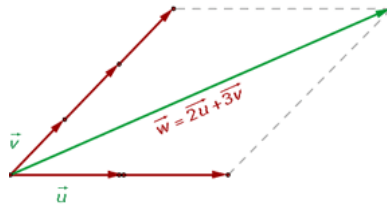
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Disclaimer: The math in this section is not too bad. The theory can be tricky to wrap your head around. PLEASE WATCH THE SUPPLEMENTARY VIDEOS THAT I HAVE POSTED!!

## 6.8 Linear Combinations and Spanning Sets

### Linear Combinations of Vectors:

For non-collinear vectors,  $\vec{u}$  and  $\vec{v}$ , a linear combination of these vectors is  $a\vec{u} + b\vec{v}$ , where  $a$  and  $b$  are scalars. The vector  $a\vec{u} + b\vec{v}$  is the diagonal of the parallelogram formed by  $a\vec{u}$  and  $b\vec{v}$ .



We have seen that any vector in  $\mathbb{R}^2$  can be written as a linear combination of  $\vec{i}$  and  $\vec{j}$  ( $\vec{OP} = (a, b) = a\vec{i} + b\vec{j}$ ), and that any vector in  $\mathbb{R}^3$  can also be written as a linear combination of  $\vec{i}$ ,  $\vec{j}$ , and  $\vec{k}$  ( $\vec{OP} = (a, b, c) = a\vec{i} + b\vec{j} + c\vec{k}$ ).

### Spanning Sets in $\mathbb{R}^2$

Because every vector in  $\mathbb{R}^2$  can be written uniquely as a linear combination of  $\vec{i}$  and  $\vec{j}$ ,  $\{\vec{i}, \vec{j}\}$  is considered a spanning set for  $\mathbb{R}^2$ . A "spanning set" refers to a vector space that contains all vectors in the plane containing the two vectors you are combining (the  $xy$  plane in the case of  $\mathbb{R}^2$ ).

In general, any two non-zero, non-collinear vectors  $\vec{u}$  and  $\vec{v}$  will span  $\mathbb{R}^2$ , and any vector,  $\vec{w}$ , in this plane can be written as a linear combination of  $\vec{u}$  and  $\vec{v}$ . That means that  $\vec{w} = a\vec{u} + b\vec{v}$  for scalars  $a$  and  $b$ . (Watch Video #1)

**Example:** Choose any two non-zero, non-collinear vectors in  $\mathbb{R}^2$ . Then write  $(-3, 5)$  as a linear combination of your two vectors.

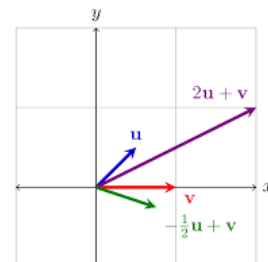


Figure 1: Vector combinations.

## Spanning Sets in $\mathbb{R}^3$

Vectors are **coplanar** if, when they are arranged tail - to - tail, their heads lie on the same plane. Three vectors are coplanar if one can be expressed as a linear combination of the other two ( $\vec{w} = \vec{a}u + \vec{b}v$ ).

Any pair of non-zero, non-collinear vectors will span a plane in  $\mathbb{R}^3$ .  
(PLEASE PLEASE PLEASE watch Video #2 to see this)

Example: Determine if the vectors  $\vec{a} = (1, 2, 3)$ ,  $\vec{b} = (2, -1, 3)$ , and  $\vec{c} = (8, 1, 5)$  are coplanar.

Example: Determine the value for  $x$  such that the points  $A(-1, 3, 4)$ ,  $B(-2, 3, -1)$  and  $C(-5, 6, x)$  all lie on a plane that contains the origin.

