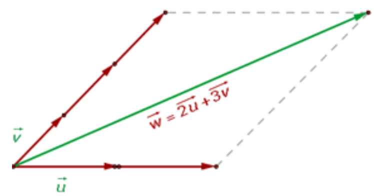
Date: ____________________

6.8 Linear Combinations and Spanning Sets

Disclaimer: The math in this section is not too bad. The theory can be tricky to wrap your head around. PLEASE WATCH THE SUPPLEMENTARY VIDEOS THAT I HAVE POSTED!!

Linear Combinations of Vectors:

For non-collinear vectors, $\vec{u}$ and $\vec{v}$, a linear combination of these vectors is $a\vec{u} + b\vec{v}$, where a and b are scalars. The vector $a\vec{u} + b\vec{v}$ is the diagonal of the parallelogram formed by $a\vec{u}$ and $b\vec{v}$.



We have seen that any vector in R^2 can be written as a linear combination of $\vec{i}$ and $\vec{j}$ ($\vec{OP} = (a, b) = a\vec{i} + b\vec{j}$), and that any vector in R^3 can also be written as a linear combination of $\vec{i}$, $\vec{j}$, and $\vec{k}$ ($\vec{OP} = (a, b, c) = a\vec{i} + b\vec{j} + c\vec{k}$).

Spanning Sets in R^2

Because every vector in R^2 can be written uniquely as a linear combination of i and j, {i, j} is considered a spanning set for R^2. A "spanning set" refers to a vector space that contains all vectors in the plane containing the two vectors you are combining (the xy plane in the case of R^2).

In general, any two non - zero, non-collinear vectors $\vec{u}$ and $\vec{v}$ will span R^2, and any vector, $\vec{w}$, in this plane can be written as a linear combination of $\vec{u}$ and $\vec{v}$. That means that $\vec{w} = a\vec{u} + b\vec{v}$ for scalars a and b. (Watch Video #1)

Example: Choose any two non - zero, non - collinear vectors in R^2. Then write (-3, 5) as a linear combination of your two vectors.

Let $\vec{a} = (-2, 4)$ and $\vec{b} = (5, -1)$

$m\vec{a} + n\vec{b} = (-3, 5)$

$m(-2, 4) + n(5, -1) = (-3, 5)$

$(-2m + 5n, 4m - n) = (-3, 5)$

① x-comp

$-2m + 5n = -3$

② y-comp

$4m - n = 5$

①×2: $-4m + 10n = -6$

⊕

$9n = -1$

$n = -\frac{1}{9}$

$-2m + 5\left(-\frac{1}{9}\right) = -3$

$-2m = -3 + \frac{5}{9}$

$-2m = -\frac{22}{9}$

$m = \frac{11}{9}$

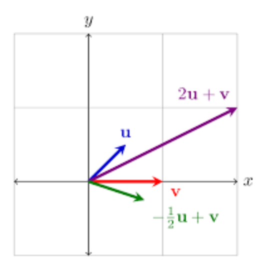


Figure 1: Vector combinations.

$\therefore \frac{11}{9}(-2, 4) - \frac{1}{9}(5, -1) = (-3, 5)$

Spanning Sets in R^3

Vectors are coplanar if, when they are arranged tail - to - tail, their heads lie on the same plane. Three vectors are coplanar if one can be expressed as a linear combination of the other two ($\vec{w} = \vec{a}u + \vec{b}v$).

Any pair of non-zero, non-collinear vectors will span a plane in R^3.
(PLEASE PLEASE PLEASE watch Video #2 to see this)

Example: Determine if the vectors $\vec{a} = (1, 2, 3)$, $\vec{b} = (2, -1, 3)$, and $\vec{c} = (8, 1, 5)$ are coplanar.

① $m(1,2,3) + n(2,-1,3) = (8,1,5)$

$(m+2n,\ 2m-n,\ 3m+3n) = (8,1,5)$

② Write 3 equations:

① $m + 2n = 8$ ② $2m - n = 1$

$2\times$② $4m - 2n = 2$

$+$

$5m = 10$

$m = 2$

$2(2) - n = 1$

$-n = -3$

$n = 3$

③ Do they work?

③ $3m + 3n = 5$

$3(2) + 3(3) \stackrel{?}{=} 5$

$6 + 9 \neq 5$

∴ Not coplanar

Example: Determine the value for x such that the points A(-1, 3, 4), B(-2, 3, -1) and C (-5, 6, x) all lie on a plane that contains the origin.

$\vec{OA} = (-1,3,4)$

$\vec{OB} = (-2,3,-1)$

$\vec{OC} = (-5,6,x)$

$m\vec{OA} + n(\vec{OB}) = \vec{OC}$

$m(-1,3,4) + n(-2,3,-1) = (-5,6,x)$

$(-m-2n,\ 3m+3n,\ 4m-n) = (-5,6,x)$

① $-m - 2n = -5$ ② $3m + 3n = 6$

②$\div 3$ $+\ m + n = 2$

$-n = -3$

$n = 3$

$-m - 2(3) = -5$

$-m = 1$

$m = -1$

Solve for x

③ $4m - n = x$

$4(-1) - 3 = x$

$\boxed{-7 = x}$