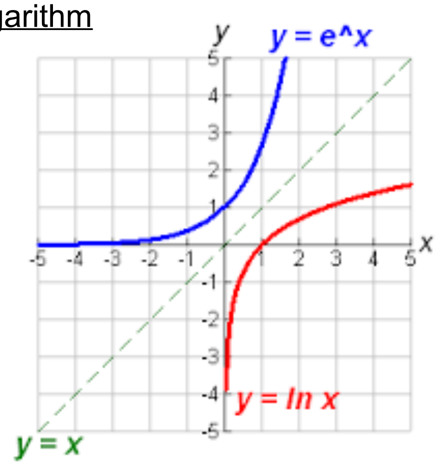
Date: ____________________

5.1 Derivatives of Exponential Functions, $y = e^x$

5.2 The Derivative of the General Exponential Function, $y = b^x$

1) Defining Euler's Number and the Natural Logarithm

- Euler's number, e, is a special irrational number like pi, and can also be called the natural number (rational approximation is e=2.718)
- The inverse of an exponential function with base e ($y = e^x$) is a logarithmic function with base e ($y=\log_e x$). This is also referred to as the natural logarithm which is represented by $y = \ln x$.



2) Derivatives of Exponential Functions of the Form $y = e^x$

Use Desmos to graph $y = e^x$ and its derivative. What do you notice?

They produce the same graph $\frac{d}{dx} e^x = e^x$

All of the derivative rules still apply. Use your previous knowledge along with your new discovery to find the derivative of the following functions:

a) $f(x) = e^{4x}$ Chain

$f'(x) = 4e^{4x}$

b) $g(x) = -3x^2(e^x + 1)$

$g'(x) = -6x(e^x+1) + e^x(-3x^2)$

$= -3x^2e^x - 6x(e^x+1)$

c) $h(x) = e^{2x}\sin x$ Product

$h'(x) = 2e^{2x}\sin x + e^{2x}\cos x$

$= e^{2x}(2\sin x + \cos x)$

d) $y = \frac{e^x - 2}{\cos x}$ Quotient

$y' = \frac{e^x(\cos x) - (e^x - 2)(-\sin x)}{\cos^2 x}$

$= \frac{e^x\cos x + e^x\sin x - 2\sin x}{\cos^2 x}$

3) <u>Finding the Derivative of a General Exponential Function, $y = b^x$</u>

For any exponential function that has a base that is a value other than e, $f(x) = b^x$, $f'(x) = b^x \ln b$. $\ln e = 1$

All derivative rules still apply when dealing with composite functions involving exponential expressions.

<u>Practice Problems:</u>

Differentiate each of the following: Product rule

1) $y = 2^x$

$y' = 2^x \ln 2$

2) $f(x) = 3x(3^x)$

$f'(x) = 3(3^x) + 3x(3^x \ln 3)$

$= 3(3^x)(1 + x\ln 3)$

3) $g(x) = 4^{2x+3}$ Chain

$g'(x) = 2(4^{2x+3} \ln 4)$

4) $h(x) = \sqrt{2^x}$

$h'(x) = \left(\frac{1}{2\sqrt{2^x}}\right)(2^x \ln 2)$

$= \frac{2^x \ln 2}{2\sqrt{2^x}}$

Let $u = 2^x$

$\frac{d\sqrt{u}}{du} = \frac{1}{2}u^{-1/2}$

$\frac{du}{dx} = 2^x \ln 2$

5) $y = 3^x - \sin x + 2\cos x$

$y' = 3^x \ln 3 - \cos x - 2\sin x$