Date: ________________

4.5 An Algorithm for Curve Sketching: A Summary

1) What information can we get from our function, $f(x)$?

- *The domain, and any discontinuities on the domain.*
- *The x - intercept(s) (let y = 0). Desmos is your friend when the zeros are at non-integer values for polynomial functions.*
- *The y - intercept (let x = 0).*
- *The equations of vertical, horizontal and oblique asymptotes, as well as the behaviour of the function as it approaches the asymptotes (use properties of limits, long division, and common factoring). Plug in values to determine if the function approaches asymptotes from above/below.*
- *The y - coordinate(s) for critical points on the function.*
- *The y - coordinate(s) for points of inflection on the function.*

2) What information can we get from the derivative, $f'(x)$?

- *The critical numbers for the function.*
- *Determine intervals of increase and decrease.*
- *Identify local maxima and minima.*

3) What information can we get from the second derivative, $f''(x)$?

- *The intervals on which a function is concave up or concave down.*
- *The locations of points of inflection.*

When you are sketching the graph of a complex function, it is important to stay organized and to understand what information is necessary. **You will not always need to use all of the tools that you have learned here**, because sometimes you can apply previous knowledge.

Example 1: Sketching an accurate graph of a polynomial function.

Sketch the graph of $f(x) = -3x^3 - 2x^2 + 5x$.

As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

① Intercepts

$f(0) = 0$

$(0, 0)$

$-x(3x^2 + 2x - 5) = 0$

$-x(3x+5)(x-1) = 0$

$(-\frac{5}{3}, 0)$, $(1, 0)$

② Critical values

$f'(x) = -9x^2 - 4x + 5$

$0 = -(9x^2 + 4x - 5)$

$= -(9x-5)(x+1)$

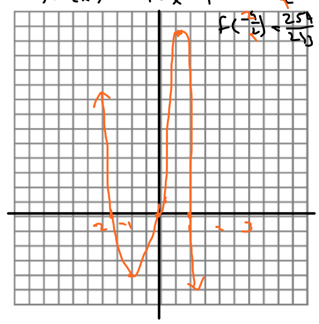
$x = \frac{5}{9}$ $x = -1$

$f(\frac{5}{9}) = \frac{300}{243}$ $f(-1) = -4$

③ Point of Inflection

$f''(x) = -18x - 4$ $x = -\frac{2}{9}$

$f(-\frac{2}{9}) = \frac{-254}{243}$



Example 2: Sketching an accurate graph of a rational function.

Sketch the graph of ~~$g(x) = \frac{x-4}{x^2-x-2}$~~. $f(x) = \dfrac{2x^2 - 3x - 5}{4x - 3}$

① Intercepts

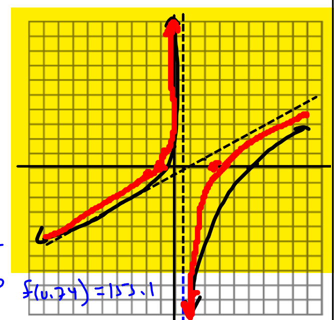
Let $x = 0$: $f(0) = \frac{5}{3}$

y-int: $(0, \frac{5}{3})$

$2x^2 - 3x - 5 = 0$

$(2x+1)(x-5) = 0$

$(-\frac{1}{2}, 0)$ + $(5, 0)$



The highlighted portions are corrected to:

① Intercepts:

$2x^2 - 3x - 5 = 0$

$(2x - 5)(x + 1) = 0$

$x = \frac{5}{2}$, $x = -1$

② Now graph in red!

② Asymptotes | Behaviour

V.A: $x = 3/4$

$\lim_{x \to \frac{3}{4}^-} f(x) = \infty$ $f(0.74) = 153.1$

Oblique: $\frac{1}{2}x - \frac{3}{8}$

$\lim_{x \to \frac{3}{4}^+} f(x) = -\infty$ $f(0.76) = -150.1$

$$\begin{array}{r} 4x-3 \,\big)\, 2x^2 - 3x - 5 \\ -\;\; 2x^2 - \frac{3}{2}x \\ \hline -\frac{3}{2}x - 5 \\ -\;\; -\frac{3}{2}x - \frac{9}{8} \\ \hline -\frac{31}{8} \end{array}$$

③ Critical Values

$f'(x) = \dfrac{(4x-3)(4x-3) - 4(2x^2-3x-5)}{(4x-3)^2}$

$= \dfrac{16x^2 - 24x + 9 - 8x^2 + 12x + 20}{(4x-3)^2}$

$= \dfrac{8x^2 - 12x + 29}{(4x-3)^2}$ ← undefined at $x = 3/4$

Critical values? QF $8x^2 - 12x + 29 = 0$ no real roots.

Interval	$x < \frac{3}{4}$	$x = \frac{3}{4}$	$x > \frac{3}{4}$
$f'(x)$	+	−	+
$f(x)$	↑	− v.a.	↑

↑ $x \in (-\infty, \frac{3}{4}), (\frac{3}{4}, \infty)$

④ Concavity

$f''(x) = \dfrac{(4x-3)^2(16x-12) - 8(4x-3)(8x^2-12x+29)}{(4x-3)^{4\,3}}$

$= \dfrac{\cancel{64x^2} - \cancel{48x} - \cancel{48x} + 36 - \cancel{64x^2} + \cancel{96x} - 232}{(4x-3)^3}$

$= \dfrac{-196}{(4x-3)^3}$ ← undef @ $x = 3/4$

Interval	$x < \frac{3}{4}$	$x = \frac{3}{4}$	$x > \frac{3}{4}$
$f''(x)$	+	−	−
$f(x)$	∪	v.a.	∩